Sprint 1

The mean of all the terms of an arithmetic sequence is the same as the mean of the first and last terms: $\frac{12+36}{2} = \frac{48}{2} = \mathbf{24}$. Another property of arithmetic sequences is that the mean is equal to the median of the set, which in this case is the middle term **24**.

Sprint 2

There are 2 rows of 9 squares each, so $2 \times 9 = 18$ squares. "Some fraction of" means to multiply by the fraction, so "$\frac{1}{2}$ of $\frac{2}{3}$ of" 18 means $\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3} \times 18 = \frac{18}{3} = \mathbf{6}$ squares are colored.

Sprint 3

The median of 7 values, arranged in increasing order, is the 4th value (3 below, 3 above), so we need the 4th smallest prime: 2, 3, 5, **7**.

Sprint 4

"Five less than a number n" means $n - 5$, and "7 more than n" means $n + 7$, so we have $|(n - 5) - (n + 7)| = |n - 5 - n - 7| = |-12| = \mathbf{12}$.

Sprint 5

With O being the origin, $\overline{\text{OA}}$ is a vertical segment of length 3 and $\overline{\text{OB}}$ is a horizontal segment of length 4. With one horizontal and the other vertical, $\overline{\text{OA}}$ and $\overline{\text{OB}}$ are perpendicular, forming the legs of right triangle AOB with $\overline{\text{AB}}$ as the hypotenuse. By the Pythagorean theorem, $\text{AB} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = \mathbf{5}$ units. You might also recognize the side lengths of this right triangle as the 3-4-5 Pythagorean triple, in which case, the hypotenuse, $\overline{\text{AB}}$ has length **5** units.

Sprint 6

The area of the regular hexagon is given to be 5. $\overline{\text{BE}}$ bisects the hexagon to form congruent trapezoids ABEF and BCDE. So, the area of trapezoid BCDE is $\frac{1}{2} \times 5 = \mathbf{2.5}$ units^2.

Sprint 7

Using the order of operations, we have $\sqrt{9^2 - 5 \times 4 + 12 \div 4} = \sqrt{(9^2) - (5 \times 4) + (12 \div 4)} = \sqrt{81 - 20 + 3} = \sqrt{61 + 3} = \sqrt{64} = \mathbf{8}$.

Sprint 8

Because $a = b$, we can substitute a for b to obtain $3a + 5 = 7a - 11$. Solving for a yields $16 = 4a$, so $a = \frac{16}{4} = \mathbf{4}$.

Sprint 9

Individually, 12 songs cost $12 \times \$0.99 = 12(\$1.00 - \$0.01) = \$12.00 - \$0.12 = \$11.88$, so buying the album saves Andy $\$11.88 - \$9.99 = \$\mathbf{1.89}$.

Sprint 10

From the first equation, we see that 3🐶 = 90, so 🐶 = 30. Substituting for 🐶 in the second equation yields 60 = 🐶 + 2🍧 = 30 + 2🍧. So, 2🍧 = 30 and 🍧 = 15. Next, substituting for 🐶 in the third equation yields 40 = 2🐶 − 🐞 = 60 − 🐞. So, 🐞 = 60 − 40 = 20. Therefore, 🐶 + 🍧 + 🐞 = 30 + 15 + 20 = **65**.

Sprint 11
At her new job, Alicia works 3 days a week for a total of 45 hours, or 45 ÷ 3 = 15 hours each workday. At her old job, she worked 4 days a week for a total of 40 hours, or 40 ÷ 4 = 10 hours each workday. The absolute difference in the numbers of hours she worked each workday at her old and new jobs is $|10-15| = \mathbf{5}$ hours.

Sprint 12
For any positive integer n, n^n is even if and only if n is even, so consider only even values of x:
$2^2 = 4$; $4^4 = 256$, so $x = \mathbf{4}$.

Sprint 13
To get the least product, we need the smallest distinct factors, thus the five least primes. Therefore,
$2 \times 3 \times 5 \times 7 \times 11 = (2 \times 5) \times (3 \times 7) \times 11 = 10 \times 21 \times 11 = \mathbf{2310}$.

Sprint 14
We know that $85\% = 0.85$, so $0.85 \times (220 - 20)\ \frac{\text{beats}}{\text{min}} = 0.85 \times 200\ \frac{\text{beats}}{\text{min}} = 85 \times 2\ \frac{\text{beats}}{\text{min}} = \mathbf{170}\ \frac{\text{beats}}{\text{min}}$.

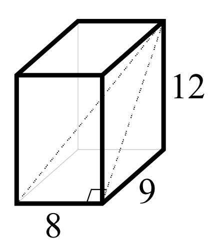
Sprint 15
For three distinct integers to have a range of 2 means the three integers must be consecutive, and the least of the three must be 1 less than the mean, so 11 – 1 = **10**.

Sprint 16
Rewriting the expressions gives us $70 = a\ \&\ 7 = 7a + 7 + a - 1 = 8a + 6$, and $a\ \&\ 3 = 3a + 3 + a - 1 = 4a + 2$. Notice that this is 1 less than half of $8a + 6$, which equals 70. So, $(4a+3) - 1 = \frac{70}{2} - 1 = 35 - 1 = \mathbf{34}$. (This method doesn't require us to find the value of a to determine the value of a & 7.)

Sprint 17
The alarm rings at $11m$ minutes after 7:00 for non-negative integers m; his dad calls at $5 + 15n$ minutes after 7:00 for non-negative integers n. We need the least non-negative integer values of m and n for which the two expressions are equal. Because times when Dad calls are at multiples of 5 min, m must be a multiple of 5. Because the coefficients of m [11] and n [15] have a least common multiple of $15 \times 11 = 165$, the time pattern repeats every 165 min, so the earliest overlap will satisfy $0 \le m < 15$. Thus, m must be 0, 5, or 10. For $m = 0$, we need $5 + 15n = 0$, which cannot be for integer n. For $m = 5$, we need $5 + 15n = 55$, which cannot be for integer n. For $m = 10$, we need $5 + 15n = 110$, for which we have a solution, $n = 7$. Therefore, the earliest overlap is 110 min after 7:00, so **8:50** a.m..

Sprint 18
Rectangle B has area $(n+5) \times n = 84$, which can be rewritten as $0 = n^2 + 5n - 84 = (n+12)(n-7)$. Therefore, $n = -12$ or $n = 7$. Only positive lengths are meaningful, so $n = 7$. The perimeter of the largest rectangle is $2\big(n + (n+3) + (n+5) + (n-2)\big) = 2(4n+6) = (8n+12) = (8 \times 7 + 12) = (56 + 12) = \mathbf{68}$ inches.

Sprint 19
The diagonal of the 9 cm × 12 cm face is the hypotenuse of a 3-4-5 right triangle scaled by a factor of 3 cm, making the hypotenuse 5 × 3 cm = 15 cm. As shown, that 15 cm face diagonal is perpendicular to the 8 cm edge, thus forming an 8-15-17 right triangle, with its hypotenuse of **17** cm being the desired diagonal.



Sprint 20
The chance of rain and Kathy being late is $0.8 \times 0.45 = 0.36$. The chance of no rain and Kathy being late is $(1 - 0.8) \times 0.3 = 0.2 \times 0.3 = 0.06$. Because there cannot simultaneously be rain and no rain, the two situations are mutually exclusive, making the total probability the sum of the two chances, $0.36 + 0.06 = 0.42 = 0.42 \times 1 = 0.42 \times 100\% = \mathbf{42}\%$.

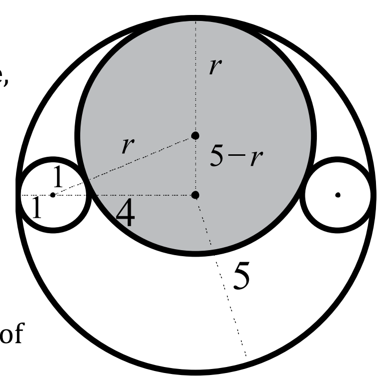
Sprint 21
The altitude to a side of a triangle is double the area enclosed by the triangle divided by the length of the side. We know the length of the sides to be $a = 11$ inches, $b = 15$ inches, $c = 16$ inches, which is not a right triangle, so the most straightforward means of determining the area is Heron's formula $\sqrt{s(s-a)(s-b)(s-c)}$, where s is the semi-perimeter $\frac{a+b+c}{2}$. Substituting, we see that $s = \frac{11+15+16}{2} = \frac{42}{2} = 21$ inches. Therefore, the area is $\sqrt{(21)(10)(6)(5)} = \sqrt{(3 \times 7)(2 \times 5)(2 \times 3)(5)} = 2 \times 3 \times 5\sqrt{7} = 30\sqrt{7}$ in^2. Thus, the altitude is $2 \times \frac{30\sqrt{7}}{15} = \mathbf{4\sqrt{7}}$ inches.

Sprint 22
All of the box sizes but one, the 10-marble box, holds a multiple of 25 marbles, so we need to decrement the total marble count of 815 successively by 10 until we reach a multiple of 25. We want the next odd multiple of 25 down from 815 to minimize the use of inefficient 10-marble boxes. That multiple is 775, with 4 of the 10-marble boxes to get there. Similarly, we need to decrement by 25 successively until we get to the next multiple of 50 down from 775, which is 750, requiring 1 of the 25-marble boxes. Again similarly, we need to decrement by 50 successively until we get to the next multiple of 100 down from 750, which is 700, requiring 1 of the 50-marble boxes. That leaves 700 marbles and boxes of 100, so we need 7 such boxes; however, we have only 5 such boxes, so we must convert the need for 2 of them to 4 boxes of 50 marbles each. Putting all this together means we have 5 boxes of 100, 5 boxes of 50, 1 box of 25, and 4 boxes of 10, and no count exceeds 5. Therefore, the minimum total needed box count is $5 + 5 + 1 + 4 = \mathbf{15}$ boxes.

Sprint 23
Since $\sqrt[n]{x} = x^{1/n}$, the equation $\sqrt[6]{x^7} - 6\sqrt[3]{x^2} = 0$ can be rewritten as $0 = x^{7/6} - 6x^{2/3} = x^{7/6} - 6x^{4/6} = x^{4/6}(x^{3/6} - 6) = x^{2/3}(x^{1/2} - 6)$. Because x is specified to be positive, we can divide out the $x^{2/3}$, leaving $x^{1/2} - 6 = 0$, so $x^{1/2} = 6$, and $x = 6^2 = \mathbf{36}$.

Sprint 24
Let $x = 0.7\overline{12}$. So, $10x = 7.\overline{12}$ and $1000x = 712.\overline{12}$. Subtracting the first equation from the second yields $1000x - 10x = 712.\overline{12} - 7.\overline{12}$, so $990x = 705$, and $x = \frac{705}{990} = \mathbf{\frac{47}{66}}$.

Sprint 25



Let r be the radius, in inches, of the shaded circle. The center of the largest circle, the center of the shaded circle, and the center of either one of the smallest circles form a right triangle, as shown, with the right angle at the center of the largest circle. The horizontal leg has length $5 - 1 = 4$; the vertical leg has length $5 - r$; the hypotenuse between the center of the shaded circle and the center of one of the smallest circles is $r + 1$. By the Pythagorean theorem, we have $4^2 + (5 - r)^2 = (r + 1)^2$. Thus, $16 + 25 - 10r + r^2 = r^2 + 2r + 1$, so $41 - 10r = 2r + 1$, and $40 = 12r$, resulting in the shaded circle having a radius of $r = \frac{40}{12} = \mathbf{\frac{10}{3}}$ inches.

Sprint 26

If the two least integers are, in increasing order of value, a and b, then the third and largest integer must be $2(a+b)+1$. The middle value, b, is the median, which is also the result of rounding the mean $\frac{a+b+(2(a+b)+1)}{3} = \frac{3(a+b)+1}{3} = a+b+\frac{1}{3}$ to the nearest integer (by dropping the $\frac{1}{3}$) to leave $a+b$. Therefore, $a+b=b$, so $a=0$, which means the value of the product of all three is **0**, no matter of the other two values.

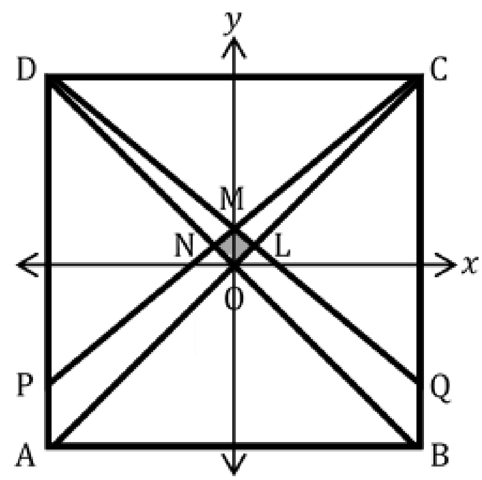
Sprint 27

Let $x=1$, and we have $p(1)=p(1)+p(2)\times 1+1^2=p(1)+p(2)+1$, so $p(2)=-1$. Now, let $x=2$, and we have $-1=p(2)=p(1)+p(2)\times 2+2^2=p(1)-2+4$, so $p(1)=-3$. Finally, let $x=5$, and we have $p(5)=p(1)+p(2)\times 5+5^2=-3-1\times 5+25=\mathbf{17}$.

Sprint 28

If we approximate $\lfloor n \rfloor$ as n, then $n^2=3$, so it would seem that n should be around $\sqrt{3}$ or $-\sqrt{3}$. However, if $1 \le n < 2$, then $\lfloor n \rfloor = 1$, so $\lfloor n \rfloor n = n < 2$ (too small) and cannot be 3. If $2 \le n$, then $\lfloor n \rfloor \ge 2$, so $\lfloor n \rfloor n$ is at least 4 (too large) and cannot be 3. Therefore, no positive n works, so n must be negative. If $-2 \le n < -1$, then $\lfloor n \rfloor = -2$, so $\lfloor n \rfloor n = -2n$, which satisfies $2 \le -2n < 4$ and is indeed 3 for $n = -\frac{3}{2} = \mathbf{-1.5}$.

Sprint 29



Draw a coordinate system, as shown, with origin O at the center of the square ABCD. The bottom of the subject quadrilateral LMNO is at the intersection of the two diagonals of the square, thus at the origin. The figure is symmetric about the y-axis, so M, too, is on the y-axis, and $\overline{MO}$ splits LMNO into two congruent triangles, MLO and MNO. Find the area enclosed by △MLO and double to obtain the shaded area. The easiest way to determine the area for △MLO is to regard $\overline{MO}$ as the base and find its length and the length of its altitude. MO is the vertical distance between $\overline{DB}$ and $\overline{DQ}$ at $x=0$. The vertical distance between $\overline{DB}$ and $\overline{DQ}$ on the left side of the square ($x=-3$) is 0 because the two segments intersect at D; we are given the vertical separation BQ on the right side of the square ($x=3$), namely 1. By linearity, the vertical distance between $\overline{DB}$ and $\overline{DQ}$ at the center (MO at $x=0$) is the average of DD and BQ, thus, $\frac{0+1}{2}=\frac{1}{2}$. Because $\overline{MO}$ is along the y-axis, the altitude to $\overline{MO}$ is the x-coordinate of L, which is the intersection of the line $\overleftrightarrow{AC}$ ($y=x$) and $\overleftrightarrow{DQ}$ (which we know now to have y-intercept $\frac{1}{2}$ at M and slope $-\frac{5}{6}$, due to rise CQ = −5 over run DC = 6, so $y=\frac{1}{2}-\frac{5}{6}x$). The intersection of these two lines at L means $x=\frac{1}{2}-\frac{5}{6}x$, so $\frac{11}{6}x=\frac{1}{2}$ and $x=\frac{3}{11}$. Therefore, the shaded area is $2\times\frac{1}{2}\times\frac{1}{2}\times\frac{3}{11}=\mathbf{\frac{3}{22}}$ units².

Sprint 30

We know that $2020 = 2^2 \times 5 \times 101$ and $2020^2 = 2^4 \times 5^2 \times 101^2$. For a, b, and c to have greatest common factor 2020, each must have at least 2 factors of 2, 1 factor of 5, and 1 factor of 101 (the exponent on each prime factor in the GCF 2020). For a, b, and c to have a least common multiple of 2020^2, each must have at most 4 factors of 2, 2 factors of 5, and 2 factors of 101 (the exponent on each prime factor in the LCM 2020^2). Moreover, for each distinct prime factor 2, 5, and 101, at least one of a, b, and c must have the least allowed exponent and at least one must have the greatest allowed exponent. This means that the exponents of 2 for (a, b, c) must be some shuffling of (2, 2, 4) with $\frac{3!}{2!\cdot 0!\cdot 1!} = 3$ [distinct] shufflings, of (2, 3, 4) with $\frac{3!}{1!\cdot 1!\cdot 1!} = 6$ shufflings, or of (2, 4, 4) with $\frac{3!}{1!\cdot 0!\cdot 2!} = 3$ shufflings; we have a total of $3 + 6 + 3 = 12$ allowed arrangements of powers of 2 among a, b, and c. The exponents of 5 for (a, b, c) must be some shuffling of (1, 1, 2) with $\frac{3!}{2!\cdot 1!} = 3$ shufflings, or of (1, 2, 2) with $\frac{3!}{1!\cdot 2!} = 3$ shufflings; that makes for a total of $3 + 3 = 6$ allowed arrangements of powers of 5 among a, b, and c. The patterns for the exponents of 101 match the patterns for exponents of 5, so there are likewise 6 allowed arrangements of powers of 101. Because the arrangements for the powers are independent with respect to the distinct prime factors, the number of ordered triples is the product of these three counts: $12 \times 6 \times 6 = \mathbf{432}$ ordered triples.

Target 1

The number of hot dogs and buns that Max needs to buy for the counts to be equal is a multiple of 6 and 8. The minimum such count that is positive is the least common multiple of 6 and 8, thus **24** hot dogs.

Target 2

Since four of the layers are stacked and one-fourth of each layer is removed, it follows that three-fourths of each layer remains. The four stacked layers have a total area of 16 units^2, so what remains after the removal is $\frac{3}{4} \times 16 = \mathbf{12}$ units^2.

Target 3

The 1 chore of laundry is to be done fourth. Any of the other 3 chores can be done first. Either of the 2 remaining chores can be done second. Then the only 1 remaining chore is done third. Thus, there are $1 \times 3 \times 2 \times 1 = \mathbf{6}$ different orders.

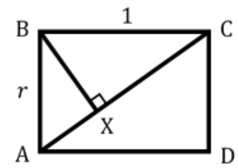
Target 4

In order for a player to score 0 points, it must be true that neither a 1 nor a 5 can be rolled, there can be no more than two of a kind, and there can be no more than two doubles. So, the only numbers that can be rolled are 2, 3, 4, and 6. Let A, B, C , and D each represent a different one of those four digits. We are looking for a roll of the form AABBCD. The number of ways to assign 2, 3, 4, and 6 to A, B, C, and D is $\frac{4!}{2!\cdot 2!} = \frac{24}{4} = 6$ ways. Each of these 6 ways has $\frac{6!}{2!\cdot 2!\cdot 1!\cdot 1!} = \frac{720}{4} = 180$ arrangements. Since there are 6^6 equally likely outcomes for a player's roll, the probability that a player scores 0 points is $\frac{6 \times 180}{6^6} = \frac{180}{6^5} = \mathbf{\frac{5}{216}}$.

Target 5

Since \$1 is equivalent to four quarters, the stack would contain $33{,}000{,}000 \times 4 = 132{,}000{,}000$ quarters. There are $12 \times 5280 = 63{,}360$ inches in a mile. Since each quarter is 0.069 inches thick, the height of the stack of quarters would be $\frac{132{,}000{,}000 \times 0.069}{63{,}360} = \mathbf{143.75}$ miles.

Target 6



Rectangle ABCD is partitioned into three right triangles as shown: △ABX (smallest), △BCX (middle-sized), and △CAD (largest), with all three similar to one another. Since the areas of the three triangles form an arithmetic progression, let the areas in increasing order be [△ABX] = $x - d$, [△BCX] = x, and [△CAD] = $x + d$. We know that $(x - d) + x = (x + d)$, so $2x - d = x + d$, and $x = 2d$. So, the triangles have areas [△ABX] = d, [△BCX] = $2d$, and [△CAD] = $3d$. That means △ABX and △BCX have areas in the ratio $\frac{[\triangle ABX]}{[\triangle BCX]} = \frac{1}{2}$, so corresponding sides AB and BC must have sides in the ratio $\frac{AB}{BC} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{\sqrt{a}}{b}$. Thus, $a = 2$, $b = 2$, and $ab =$ **4**.

Target 7

For the conversion formula $F = \frac{9}{5}C + 32$, we want to know what Celsius temperature satisfies $F = -C$. Therefore, substitute $-C$ for F in the conversion formula to obtain: $-C = \frac{9}{5}C + 32$, which reduces to $\frac{14}{5}C = -32$, so $C = \frac{5}{14}(-32) = -\frac{160}{14} = -11.428\ldots$, which expressed to the nearest tenth is **−11.4** °C.

Target 8

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Square #1 shares a side with only two other squares, #2 and #5, so all three must be the same color in order to have #1 the same color as exactly two squares it shares a side with. Likewise, #3, #4, and #8 must be the same color (but not necessarily the same color as #1, #2, and #5); #9, #13, and #14 must be the same color; #12, #15, and #16 must be the same color. Square #2 shares a side with #1 (already known to be the same color), #3, and #6, so exactly one of #3 and #6 must be the same color. Let's try #3 first, which makes all of #1 through #5 and #8 the same color, but #6 a different color. Now, #6 has only #7 and #10 to be the same color, and then #7 needs #11 to be the same color, making the center section of #6, #7, #10, and #11 all the same color. That leaves #8 with only #12 available to go with #4 being the same color. The process works its way around the outer edge so that all squares on the outer edge must be the same color. There are 4 options for the color of the outer edge and 3 remaining color options for the interior region, making a total of <u>12</u> option pairs for this configuration. Let's now try #6 instead of #3 as being the same color as #2, which makes the upper left quadrant all one color. Squares #2, #6, and #5 are a full cadre of neighbors of the same color, so #3, #7, #9, and #10 must have different colors than the squares in the upper left quadrant. Square #3 needs #7 to join with #4 and #8 to be the same color, making the upper right quadrant all one color. Similar analysis shows the lower left quadrant to be all one color and the lower right quadrant all one color. We can have all 4 quadrants different colors for $4! =$ <u>24</u> options for this configuration; we can instead have the upper left and lower right quadrants the same color and the upper right and lower left quadrants the same color but different from the other pair, again making $4 \times 3 =$ <u>12</u> options for this configuration. Lastly, we can have one pair of opposite quadrants the same color and the remaining two quadrants two different colors; there are 2 options for which pair of quadrants are the same color, 4 options for which color that pair will be, 3 options for the color of one of the remaining quadrants, and 2 options for the color of the last quadrant, making $2 \times 4 \times 3 \times 2 =$ <u>48</u> total options for this configuration. The total number of ways to color the squares is $12 + 24 + 12 + 48 =$ **96** ways. One sample for each configuration is:

R	R	R	R
R	B	B	R
R	B	B	R
R	R	R	R

12 color options

R	R	B	B
R	R	B	B
Y	Y	G	G
Y	Y	G	G

24 color options

R	R	B	B
R	R	B	B
B	B	R	R
B	B	R	R

12 color options

R	R	B	B
R	R	B	B
Y	Y	R	R
Y	Y	R	R

48 color options