
MATHCOUNTS®

2020
■ State Competition ■
Countdown Round
Problems 1–80

This booklet contains problems to be used
in the Countdown Round.

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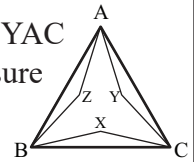
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1. _____ The arithmetic mean of Josh's four test scores is 80. If three of Josh's test scores have a mean of 87, what is Josh's score for the fourth test? Express your answer to the nearest whole number.
2. _____ What is the value of the expression $\sqrt{3^4 + 3^4 + 3^4 + 3^4}$?
3. _____ What is the sum of the integer solutions of the inequality $|5x - 3| \leq 8$?
4. _____ What is the value of $\frac{7 + \frac{4}{9}}{9 + \frac{4}{7}}$? Express your answer as a common fraction.
5. _____ (unit cubes) The top and bottom faces of a large cube are painted orange, and the four lateral faces are painted purple. The large cube is then cut into 125 unit cubes. How many of the unit cubes have exactly one purple face?
6. _____ (years) The sum of Andrea's age and her younger sister's age is 25. If the product of their ages is 150, how many years older than her sister is Andrea?
7. _____ (minutes) The speed of light in space is 299,792.46 km/s. How many minutes does light take to travel 54,600,000 km from Mars to Earth? Express your answer to the nearest whole number.
8. _____ A list of four single-digit positive integers has median m and unique mode d . What is the greatest possible value of $m - d$? Express your answer as a decimal to the nearest tenth.
9. _____ (students) Every student at Bo Jackson High School plays football, baseball or both football and baseball. Half of the football players also play baseball, and twice as many students play baseball as play football. If 200 students play both sports, how many total students attend Bo Jackson High School?
10. _____ (units²) What is the area, in square units, of a right triangle that has integer side lengths and perimeter 36 units?
11. (Day) _____ A town in Fantasyland had a high temperature of 60 degrees on Day 1. Every day thereafter, the high temperature was 4 degrees lower than it was the previous day. On what day was the high temperature 8 degrees?
12. _____ (units) In equilateral triangle ABC, the length of the angle bisector from vertex A to side BC is 12 units long. What is the perimeter of the triangle, in units? Express your answer in simplest radical form.
13. _____ If $\sqrt{8} + \sqrt{18} = \sqrt{x}$, what is the value of x ?

14. _____ (degrees)

Equilateral triangle ABC contains congruent isosceles triangles ZAB, YAC and XBC as shown. If $m\angle ZAB = 13$ degrees, what is the degree measure of $\angle YAZ$?



15. _____

If $2A2,7B5$ is divisible by 99, for digits A and B, what is the value of $10A + B$?

16. _____

What is the least positive odd number that has a remainder of 2 when it is divided by 3 and that has a remainder of 3 when it is divided by 5?

17. _____ (ordered triples)

How many ordered triples of positive integers (x, y, z) satisfy $xyz = 12$?

18. _____ (miles)

Melissa runs a mile in 8 minutes 30 seconds. At this rate, how many full miles will she run in 45 minutes?

19. _____

What is the arithmetic mean of all two-digit numbers whose digits sum to 9?

20. _____ (cents)

Ali has five coins totaling less than \$1.00, with no more than three of any type of coin. No combination of four of his coins is enough to purchase an apple for 83 cents. If Ali is able to purchase the apple using all five coins, how many cents worth of change should he get?

21. _____ (cm)

A certain right triangle has one leg of length 30 cm and a hypotenuse of length 34 cm. How many centimeters long is the other leg?

22. _____

If x is a number such that $x^2 + x - 1 = 0$, what is the value of $4x^2 + 4x + 1$?

23. _____ (fabric squares)

Rachel is decorating a rectangular box that measures 12 inches by 16 inches by 8 inches. She plans to cover five of the six faces of the box with congruent non-overlapping fabric squares. What is the fewest whole fabric squares that Rachel needs to cover the box as described?

24. _____

What is the value of $8100 - (94 \times 86)$?

25. _____

Trajan randomly selects three distinct numbers from the set $\{1, 2, 3, 4, 5\}$. What is the probability that the sum of the selected numbers is divisible by 3? Express your answer as a common fraction.

26. _____

What is the remainder when $50!$ is divided by 51?

27. _____

One-half of two-thirds of one-fourth of some number is 11. What is the number?

28. _____ (ways)

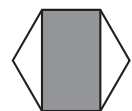
Six students will form a 3-person red team and a 3-person blue team to play a game. In how many ways can the teams be formed if two of the students, Eric and David, cannot be on the same team?

29. _____ (dollars)

A lawn measuring 30 feet by 60 feet is to be covered with grass turf that costs \$40 per square yard. How many dollars will it cost to cover the entire lawn with grass turf?

30. _____ (ways) How many distinct ways are there to arrange the seven letters in ALGEBRA?
31. _____ If x and y are positive numbers such that 20% of $x\%$ of y is equal to x , what is the value of y ?
32. _____ (miles) To get to the grocery store from her apartment, Kathy walks one block south, then six blocks west, then nine blocks north. A block is 0.1 mile. If Kathy could walk in a straight line from her apartment to the store, how many miles would the trip be?
33. _____ (cm³) What is the volume of a right circular cone of height 4 cm that is similar to a cone that has radius 3 cm and volume 6π cm³? Express your answer in terms of π .
34. _____ What is the sum of the solutions of the equation $4x^2 = 12x + 7$?
35. _____ An integer is called *eclectic* if all its digits are distinct. What is the sum of the greatest 3-digit eclectic integer and the least positive 3-digit eclectic integer?
36. _____ Sky attempted to score a goal five times. For each attempt, she was equally likely to make a goal or miss. What is the probability that Sky scored at least two goals? Express your answer as a common fraction.
37. _____ (units²) Two opposite vertices of a square have coordinates (3, 5) and (−11, 7). What is the area of this square, in square units?
38. _____ (ft³) How many total cubic feet of glass are needed to make a 4-foot by 6-foot window that is $\frac{1}{8}$ inch thick? Express your answer as a common fraction.
39. _____ (m/s) Agent 020 chases a fugitive who is initially 600 meters ahead of her. The fugitive runs at a constant speed of 4 m/s. Agent 020 also runs at a constant speed, and takes 5 minutes to catch the fugitive. What is Agent 020's speed, in meters per second?
40. _____ If $3^k \times 9^2 \times 27 = 3^{13}$, what is the value of k ?
41. _____ What is the closest integer to the fourth root of 4000?
42. _____ (ways) In how many distinct ways can regular pentagon ABCDE be constructed so that each side is either red or blue and no pair of adjacent sides are the same color?
43. _____ (cents) Rae pays \$4.69 for a six-inch sub sandwich, and Elijah pays \$8.30 for a foot-long sub sandwich. How many more cents per inch of sub sandwich does Rae pay than Elijah?
44. _____ What is the median of the set of all positive divisors of 2020? Express your answer as a decimal to the nearest tenth.

45. _____ (units) In triangle ABC, $AB = 10$ units and $BC = 16$ units. What is the sum of the least and greatest possible integer lengths for side AC?
46. _____ The sum of the first five terms of an arithmetic sequence is 15. What is the third term of this sequence?
47. _____ What is the value of $123.45 + 234.51 + 345.12 + 451.23 + 512.34$? Express your answer as a decimal to the nearest hundredth.
48. _____ (ways) How many ways are there to make exactly 50 cents using quarters, dimes and nickels if at least one of each type of coin must be used?
49. _____ What is the value of $111^2 - 11^2 + 1^2$?
50. _____ Including Gene, 10 individuals audition for the three available parts in a new musical. If everyone is equally likely to be chosen for any part, but no one can be chosen for more than one part, what is the probability that Gene gets a part in the musical? Express your answer as a common fraction.
51. _____ (in²) What is the area, in square inches, of an isosceles trapezoid with bases of lengths 6 inches and 14 inches and nonparallel sides of length 5 inches?
52. _____ What is the quotient when 1,000,000,000 is divided by $2^8 \cdot 5^7$?
53. _____ If $f(x, y) = \sqrt{y + \sqrt{x + \sqrt{y}}}$, what is the value of $f(116, 25)$?
54. _____ What is the value of $\frac{10^7 - 10^6}{18}$?
55. _____ (amounts) Elias has four \$100 bills, three \$10 bills and six \$1 bills. How many different amounts of money can Elias make using some or all of these bills?
56. _____ What is the ratio of the area of the gray region to the total area of the regular hexagon shown here? Express your answer as a common fraction.
57. _____ (minutes) James can eat 12 jelly beans per minute, and Jenny can eat 8 jelly beans per minute. How many minutes will it take Jenny and James to eat a combined total of 100 jelly beans?
58. _____ The arithmetic mean of seven consecutive integers is x . What is the absolute difference between the greatest of the seven integers and x ?
59. _____ Andrea lists six prime numbers whose sum is a multiple of 10. Andrea's numbers, in increasing order, are 13, 29, 31, p , 47, 53. What is the value of p ?



60. (ordered pairs) For how many ordered pairs of integers (x, y) is the point (x, y) exactly 5 units away from the point $(20, 15)$?

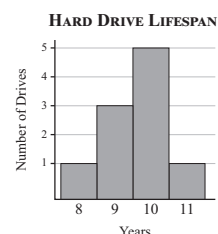
61. If the lines defined by the equations $3x + 2y = 9$ and $bx - y = 119$ are parallel to each other, what is the value of b ? Express your answer as a common fraction.

62. (meters) A certain rectangular prism has two faces, each of area 42 m^2 , two faces, each of area 54 m^2 and two faces, each of area 63 m^2 . What is the sum of the 12 edge lengths of the prism, in meters?

63. If $5^{x^3-8} + 9 = 10$, what is the value of x ?

64. What is the value of $\sqrt{\frac{1}{36} + \frac{1}{64}}$? Express your answer as a common fraction.

65. (years) Jimmy tested the lifespans, in years, of several hard drives. His results are given in the histogram shown. How many years is the average lifespan of the hard drives Jimmy tested? Express your answer as a decimal to the nearest tenth.



66. (mi/h) Jenny ran 1 mile up a hill in 15 minutes. If it took her 9 minutes to run back down the hill, along the same path, what was Jenny's average speed, in miles per hour, over the whole run?

67. Define the binary operation \square as $a \square b = a^2 + 2ab + b^2$. What is the value of $(1 \square 1) \square 9$?

68. What is the value of $\frac{3^{2019} + 3^{2016}}{3^{2018} - 3^{2015}}$? Express your answer as a common fraction.

69. The numbers $\{6, 5, 3, 7, y\}$ have arithmetic mean y . What is the median of these numbers? Express your answer as a decimal to the nearest hundredth.

70. (years) A giant sequoia tree measures 246 feet tall, and a nearby California redwood tree measures 278 feet tall. If each tree continues to grow exactly one foot per year, in how many years will the California redwood be exactly 10% taller than the giant sequoia?

71. A special machine has an input slot and an output slot, such that for every real number, x , if $5x + 8$ is dropped into the input slot, $3x + 1$ will be the output. If the input is -12 , what number will be the output?

72. What is the value of $\sqrt{6! + 3! + 2! + 1!}$?

73. _____ Circle A has area 16 units^2 and circle B has area 64 units^2 . What is the ratio of the radius of circle A to the radius of circle B? Express your answer as a common fraction.
74. _____ (dollars) In 2017, Jonah's income was \$57,500. His income increased by 400% in 2018, but then decreased by 80% in 2019. What was Jonah's income in 2019, in dollars?
75. _____ What is the remainder when $1 + 3 + 3^2 + 3^3 + \dots + 3^{2020}$ is divided by 81?
76. _____ Together, the total value of $2n$ nickels, $5n$ pennies and n dimes is \$1.50. What is the value of n ?
77. _____ A magic square is a square in which the sum of the numbers in each row, each column and each diagonal is the same. What is the value of $a + k$ in the magic square shown?
- | | | |
|-----|-----|-----|
| x | 3 | 8 |
| 9 | a | y |
| b | c | k |
78. _____ The sum of the digits of a particular three-digit number is 16. The tens digit of this number is three times the units digit, and the hundreds digit is one more than the units digit. What is the number?
79. _____ If a and b are positive integers with $a^b = 64$, what is the sum of all possible values of b ?
80. _____ What is the value of $2020^2 - 2020 \times 2019 + 2019^2 - 2019 \times 2020$?