
MATHCOUNTS®

2019
■ State Competition ■
Team Round
Problems 1–10

School _____
Chapter _____
Team _____, Captain
Members _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

| Total Correct | Scorer's Initials |
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1. _____ If n is a number such that $n^{12} = 216$, what is the value of n^4 ?

2. _____ What is the smallest prime number p for which $2^p - 1$ is composite?

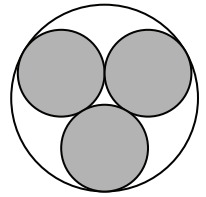
3. _____ The third and fourth terms of a geometric sequence are 100 and 200. What is the first term of the sequence?

4. _____ years Suppose that Martians have eight fingers and use a base-eight (octal) number system. If Marty the Martian says he is 37 years old on Mars, how old is he in Earth's base-ten system?

5. _____ M&Ms Mr. Schwin has a large jar containing M&Ms, each with the letter "m" stamped on it. He removes 1000 candies from the jar, and removes the letter "m" from each one. He then returns all of the M&Ms to the jar. After thoroughly mixing up the candies in the jar, he randomly removes 1000 candies from the jar and finds that 245 of them do not contain the letter "m". What is the expected number of M&Ms in the jar? Express your answer to the nearest whole number.

6. _____

In the figure, three congruent, non-overlapping shaded circles are inside a large circle. The greatest possible value of the ratio of the total area of the shaded regions to the area of the large circle can be expressed in the form $a + b\sqrt{c}$. What is the value of $a + b + c$?



7. _____

If a and b are real numbers with $a - b = 1$, what is the least possible value of $a^5 - b^5$? Express your answer as a common fraction.

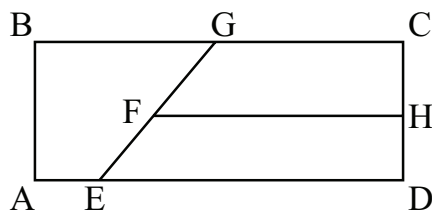
8. _____

If each question mark in the equation shown is replaced with an exponent of either 1 or -1 , what is the least possible value of $|x - 1|$? Express your answer as a common fraction.

$$x = 1^? \times 2^? \times 3^? \times 4^? \times 5^? \times 6^? \times 7^? \times 8^? \times 9^?$$

9. _____ units

In rectangle ABCD, $AB = 4$ units and $BC = 12$ units. Point E is on side AD such that $AE = 2$ units. Segment EG is drawn from point E to point G on side BC, as shown, so that the area of trapezoid ABGE is one-third that of rectangle ABCD. A segment is then drawn parallel to side BC from point F on segment EG to point H on side CD so that trapezoids FGCH and EFHD have equal area. What is the length of segment FH? Express your answer in simplest radical form.



10. _____ %

Amy and Rob are playing a dice game using a 10-sided fair die, with sides numbered 1 through 10. Rob earns 3 points if the number rolled is prime, and Amy earns 2 points if the number rolled is even. After 4 rolls, what is the percent probability that Amy has won more points than Rob? Express your answer to the nearest hundredth.