0
1
2
3
4
5
6
7
8
9

MATHCOUNTS®

2019
■ Chapter Competition ■
Team Round
Problems 1–10

School ______________________________

Team Members ______________________________, Captain

____

____

____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 10 problems which the team has 20 minutes to complete. Team members may work together in any way to solve the problems. Team members may talk to each other during this section of the competition. This round assumes the use of calculators, and calculations also may be done on scratch paper, but no other aids are allowed. All answers must be complete, legible and simplified to lowest terms. The team captain must record the team's official answers on his/her own competition booklet, which is the only booklet that will be scored. If the team completes the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

National Sponsors
Raytheon Company
U.S. Department of Defense
Northrop Grumman Foundation
National Society of Professional Engineers
CNA Insurance
Texas Instruments Incorporated
3Mgives
Phillips 66
Art of Problem Solving
NextThought

Raytheon

2019 MATHCOUNTS
National Competition Sponsor

Founding Sponsors: National Society of Professional Engineers, National Council of Teachers of Mathematics and CNA Foundation

1. ________ inches

Dara creates three congruent isosceles triangles, each of which has two legs of length 7 inches and a base of length 5 inches. She joins these triangles together, gluing an entire side of one triangle to an entire side of another triangle, until she has used all three triangles to form a single polygon. What is the greatest possible perimeter of the polygon?

2. ________

For what value of n is the equation $5^{50} = n^n$ true?

3. ________

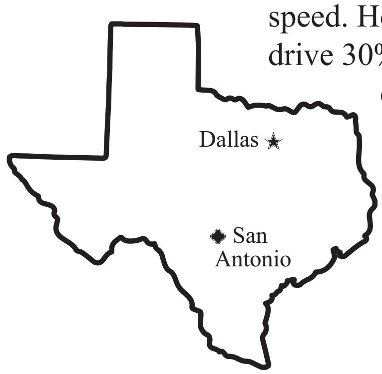
Lior brought two-thirds of his candy bar to school to share with two friends. Celine took one-fourth of the shared piece and Elliot took half of what was left. The final portion of that piece was split equally among Lior, Celine and Elliot. What fraction of Lior's original candy bar did Celine get? Express your answer as a common fraction.

4. ________ $units^2$

When a triangle with vertices at (2, 0), (10, 2) and (6, 6) is rotated 360 degrees about the point (0, −3), the sides and vertices of the triangle sweep out a region in the shape of an annulus (ring). What is the area of the annulus? Express your answer in terms of π.

5. ________

On Monday, Crystal drives from San Antonio to Dallas at a constant speed. On her return trip on Tuesday, she initially drives 10% faster than her Monday speed. However, during the last part of her trip, she runs into traffic and has to drive 30% slower than her Monday speed. As a result, her Tuesday trip takes exactly as long as her Monday trip. What fraction of the distance of her Tuesday trip does she travel at the faster speed? Express your answer as a common fraction.



6. ______________

In the 6-by-6 grid shown, each square contains a different positive integer. Hiroshi placed coins on some squares in the grid. No square had more than one coin, and the total number of coins in each row and in each column is indicated by the row and column labels in **bold**. What is the sum of the numbers in the squares on which Hiroshi placed a coin?

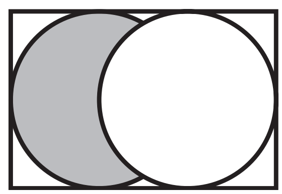
	2	**2**	**1**	**5**	**3**	**5**
3	1	2	3	4	5	6
4	7	8	9	10	11	12
3	13	14	15	16	17	18
2	19	20	21	22	23	24
4	25	26	27	28	29	30
2	31	32	33	34	35	36

7. ______________

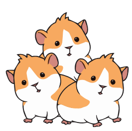
Shi Yin chooses four numbers at random, without replacement, from the set of integers from 1 to 10, inclusive. What is the probability that the product of these four numbers has units digit 0? Express your answer as a common fraction.

8. ______________

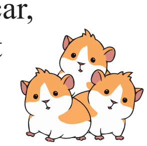
Two congruent circles are inscribed in a 12×18 rectangle such that each circle is tangent to three of the four edges of the rectangle as shown. What is the probability that a randomly chosen point in the rectangle is in the shaded area? Express your answer as a decimal to the nearest hundredth.



9. ______________ ways



Jane has six different hamsters for which she has two cages, one red and one blue. She wants to put three hamsters in the red cage and three in the blue cage, but two of the hamsters, Felix and Oscar, do not get along and cannot be in the same cage. In how many different ways can she choose which three hamsters to put in the red cage?



10. ______________

Nala draws triangle ABC. She randomly chooses a real number a between 0 and 1. She then picks point P on $\overline{AB}$ such that $\frac{PB}{PA} = a$. Point Q is chosen on $\overline{AC}$ such that the area of triangle APQ is half the area of triangle ABC. What is the probability that $\frac{QC}{QA} \le \frac{1}{4}$? Express your answer as a common fraction.