MATHCOUNTS Minis September 2012 Activity Solutions

Warm-Up!

1. Armond first travels 10 feet at a rate of 1 foot every 10 seconds, then travels 10 feet at a rate of 1 foot every 15 seconds. That means it takes Armond 10 × 10 = 100 seconds to travel from the wall to the crumb, and another 10 × 15 = 150 seconds to travel back to the wall carrying the crumb. Therefore, the entire trip takes Armond 100 + 150 = 250 seconds, which is equivalent to 250 ÷ 60 = $\mathbf{4\frac{1}{6}}$ minutes.

2. Since it takes 80 seconds for the 1-mile long train to pass a signpost, the train is traveling at a rate of 1 mile every 80 seconds. There are 60 × 60 = 3600 seconds in one hour, so the train will travel 3600 ÷ 80 = **45** miles in one hour.

3. Since the trip takes 24 minutes at a rate of 30 miles per hour, to make the trip in 12 minutes, which is half the time, the rate needs to be doubled. Therefore, to complete the same trip in 12 minutes, the rate of travel would need to be 30 × 2 = **60** miles per hour.

4. We are told that Fred runs 400 yards in 50 seconds, which is a rate of 8 yards per second. Since there are 6 × 60 = 360 seconds in 6 minutes, it follows that Fred will run 8 × 360 = 2880 yards in 6 minutes. This is equivalent to 2880 ÷ 1760 = $\mathbf{1\frac{7}{11}}$ miles.

The Problem is solved in the MATHCOUNTS Mini.

Follow-up Problems

5. Since Jayne rides her bike at a rate of 12 miles per hour, after 30 minutes, or one half hour, she travels 6 miles. Since 50% of Jayne's initial rate is 6 miles per hour, it follows that an increase of 50% results in the increased rate of 12 + 6 = 18 miles per hour. Since Jayne rides her bike at a rate of 18 miles per hour for the second half hour, she travels 9 miles. Therefore, Jayne rides her bike a total of 6 + 9 = **15** miles.

6. Let's employ a method similar to the one used in the video. Since Jill drives for 80%, or 4/5 the time that Jack drives the same distance, Jill's rate must be 5/4 Jack's rate. If we let r represent Jack's rate, we can write Jill's rate as $(5/4)r$. We also are told that Jill travels 10 miles per hour faster than Jack. So Jill's rate also can be written as $r + 10$. These quantities are equivalent so we can write the equation $(5/4)r = r + 10$. Solving for r, we get $(1/4)r = 10 \rightarrow r = 40$. Therefore, Jack traveled at an average rate of **40** miles per hour.

7. Since Tirunesh travels at a rate of 8 meters per second, and Sally travels at a rate of 7 meters per second, it follows that Tirunesh travels 1 meter farther than Sally each second. So the first time they meet is after Tirunesh has traveled 400 meters more than Sally has traveled. Traveling 1 meter per second faster than Sally, it will take **400** seconds for Tirunesh to travel 400 meters farther than Sally.

8. This time Tirunesh and Sally are traveling in opposite directions at a combined rate of 8 + 7 = 15 meters per second. At this rate, they will first cover a combined distance of 400 meters after 400 ÷ 15 = 80/3 seconds. Since 5 minutes is equivalent to 5 × 60 = 300 seconds, together Tirunesh and Sally will cover a combined distance of 400 meters 300 ÷ (80/3) = 45/4 = 11.25 times. That means Tirunesh and Sally will give each other **11** high fives.