MATHCOUNTS Minis **April 2012 Activity Solutions**

Warm-Up!

1. We are asked to determine the total number of sequences of the five coin flips that result in 4 heads (*H*) and 1 tail (*T*). If we think of the results filling five spaces, one such sequence would be:

H *H* *H* *H* *T*

The only thing that differentiates sequences of the 4 *H*s and 1 *T* is the location of the *T*. Since the *T* can be in one of five locations, there are **5** different sequences: *HHHHT*, *HHHTH*, *HHTHH*, *HTHHH* and *THHHH*.

2. We are asked to determine the total number of sequences of the five coin flips that result in 3 heads and 2 tails. As with the previous problem we can consider the different locations of the two *T*s to determine the different sequences. Each of the two *T*s will fill one of the five spaces, so that's 5 choose 2. Therefore, there are ${}_5C_2 = 5!/(3!\,2!) = (5 \times 4)/2 = 20/2 = 10$ ways to place the 2 *T*s, so there are **10** different sequences.

3. Again, to determine the number sequences for randomly selecting four purple (*P*) marbles and two green (*G*) marbles, we can consider the different locations of the two green marbles in the sequence. Each of the two *G*s will fill one of six spaces, so that's 6 choose 2. Therefore, there are ${}_6C_2 = 6!/(4! \cdot 2!) = (6 \times 5)/2 =$ **15** different sequences.

4. If the probability of this coin coming up heads is twice as likely as it coming up tails, we have that $P(H) = 2(P(T))$. Since $P(H) + P(T) = 1$, we have $2(P(T)) + P(T) = 1 \rightarrow 3(P(T)) = 1$, and $P(T) = 1/3$. That means that $P(H) = 2/3$. There are four ways for this coin to come up with 3 *H*s and 1 *T* in four flips: *HHHT*, *HHTH*, *HTHH* and *THHH*. The probability of each of these sequences occurring is the same, and each one is $(2/3)^3 \times (1/3) = 8/81$. Therefore, the probability of getting 3 *H*s and 1 *T* when this particular coin is flipped four times is $4 \times (8/81) =$ **32/81**.

The Problem is solved during the MATHCOUNTS Mini.

Follow-up Problems

5. We are asked to determine the probability of pulling 3 red (*R*) and 2 orange (*O*) balls when five balls are pulled randomly, with replacement, from the bag of 15 balls. The probability of randomly pulling a red ball is $P(R) = 5/15 = 1/3$, and the probability of pulling an orange ball is $P(O) = 1 - 1/3 = 2/3$. Thus, the probability of *RRROO* is $(1/3)^3 \times (2/3)^2 = 4/243$. But there are ${}_5C_3 = 5!/(2! \cdot 3!) = (5 \times 4)/2 = 20/2 = 10$ ways to get 3 *R*s and 2 *O*s. Therefore, the probability of pulling exactly 3 red balls randomly, with replacement, is $10 \times (4/243) =$ **40/243**.

6. We are told that when playing the Yankees, the probability of a Phillies win (*P*) is 2/3. So the probability of a Yankees win is $P(Y) = 1 - 2/3 = 1/3$. We are asked to determine the probability the Phillies will win the series 4 games to 2. The probability of *YYPPPP* is $(1/3)^2 \times (2/3)^4 = 16/729$. Since we are told the series must be exactly six games, the fourth win of the Phillies must be the sixth game. If their fourth win were to occur before the sixth game, no more games would be played following that win. So in a series of six games where the sixth game must be won by the Phillies, there are ${}_5C_2 = 5!/(3! \cdot 2!) = (5 \times 4)/2 = 10$ ways for this to occur. Therefore, the probability the Phillies will win the series in 4 games to 2 is $10 \times (16/729) =$ **160/729**.

7. We are asked to determine the number of ways to arrange the letters in the word BANANA. If we think of the letters filling six spaces, there are 6! ways to arrange the six letters of the word in the six spaces. But we must account for the fact that the As are identical and the Ns are identical. We would have counted 3! arrangements of the As and 2! arrangements of the Ns. Dividing we see that the number of different arrangements of the six letters is $6!/(3! \cdot 2!) = (6 \times 5 \times 4)/2 = 120/2 =$ **60** arrangements.

8. We are told that the probabilities of Alice, Bob and Carol each winning a game of chess is $P(A) = P(B) = P(C) = 1/3$. We are asked to determine the probability that Carol will win the championship in exactly 6 games. The only way Carol can win the championship after exactly 6 games have been played is if the third game she wins is the sixth game played. If her third win were to occur before the sixth game, no more games would be played following that win. We also know that the other three games can't be won by the same person, otherwise they would win the championship. So there must be two wins for Alice and one win for Bob, or one win for Alice and two wins for Bob. So the possible wins for the first five games are *ABBCC* or *AABCC*. Using the methodology from the video we see there are $5!/(2! \cdot 2!) = (5 \times 4 \times 3)/2 = 60/2 =$ 30 orders in which the wins *ABBCC* (in any order) can occur and 30 orders in which the wins *AABCC* (in any order) can occur. That's a total of 60 sequences for the possible wins of the first five games. Since the probability of any of the three players winning each of those five games is 1/3, the probability that Carol will win the championship in exactly 6 games is $60 \times (1/3)^5 = 60/729 =$ **20/243**.