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Activity Sheet for the March, 2012, MATHCOUNTS Mini

Warm-Up!

Try these problems before watching the lesson.

1. Find the missing terms in each of the following arithmetic sequences:

   (a) $5, 11, \_\_\_, \_\_\_, \_\_\_$
   (b) $\_\_\_, \_\_\_, \_\_\_, 5, 11$
   (c) $5, \_\_\_, \_\_\_, \_\_\_, 11$

2. Find the missing terms in each of the following geometric sequences:

   (a) $2, 18, \_\_\_, \_\_\_, \_\_\_$
   (b) $\_\_\_, \_\_\_, \_\_\_, 2, 18$

3. The third and fourth terms of an arithmetic sequence are the first and second terms of a geometric sequence. If the first two terms of the arithmetic sequence are $5, 2$, then what is the fourth term of the geometric sequence?

4. The first two terms of an arithmetic sequence are 4, 10. What is the first term greater than 1000?

The Problem

First Problem: The 80th term of an arithmetic sequence is twice the 30th term. If the first term of the sequence is 7, what is the 40th term?

Second Problem: The numbers a, b, c, and d form a geometric sequence, in that order. If b is three more than a, and c is nine more than b, what is the value of d?

Third Problem: The geometric mean of two positive numbers a and b is $\sqrt{ab}$. The third term of an arithmetic sequence of positive numbers, in which the difference between the terms is not zero, is the geometric mean of the first and eleventh terms. What is the ratio of the second term to the first term of the sequence?

Follow-up Problems

5. If the second term of an arithmetic sequence is -7 and the fifth term is 106, then what is the fourteenth term?

6. The third term of a geometric sequence of negative numbers is -2 and the fifteenth term is -162. What is the sixth term of the sequence?

7. An amoeba is placed in a puddle one day. Every day, each amoeba splits into two amoebas, and each of the new amoebas immediately grows to the same size as the original amoeba. The puddle is finally completely full of amoebas on the 23rd day. What was the first day on which the puddle was half-full of amoebas?

8. The first term of an arithmetic sequence is 1, another term of the sequence is 91 and all of the terms of the sequence are integers. How many distinct arithmetic sequences meet these three conditions?

9. Let v, w, x, y, z be an arithmetic sequence. If $\frac{y}{w} = 3$, then what is $\frac{z}{x}$?

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).