MATHCOUNTS Minis **February 2012 Activity Solutions**

**Warm-Up!**

1. Let's first consider the probability that when flipping a fair coin 3 times the total number of heads (H) is greater than the total number of tails (T). The possible outcomes when flipping a coin 3 times are HHH, HHT, HTH, THH, TTT, TTH, THT, HTT. Notice that in half of these outcomes the total number of heads is greater than the total number of tails. This will be the case each time the coin is flipped an odd number of times. Therefore, the probability of the total number of heads being greater than the total number of tails when flipping a fair coin 37 times is **1/2**.

2. To determine the total number of times "Hi!" is spoken we need to determine the number of multiples of 2, the number of multiples of 3 and the number of multiples of 4 between 1 and 100, inclusive. We know that half of the numbers from 1 to 100 are even and thus divisible by 2. So 50 of the numbers Abby counts aloud are multiples of 2. Then we know that half of those 50 numbers are divisible by 4. Therefore, 25 of the numbers Abby counts aloud are multiples of 4. Every third number from 1 to 100 is divisible by 3, for a total of 33 multiples of 3. It follows that "Hi!" will be spoken 50 + 25 + 33 = **108** times.

3. The 2-digit palindromes are 11, 22, 33, 44, 55, 66, 77, 88 and 99, which can be written 10 + 1, 20 + 2, 30 + 3, and so forth. The sum of the units digits is 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45. The sum of the numbers represented by the tens digits is 10(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) = 10(45) = 450. Therefore, the sum of the 2-digit palindromes is 45 + 450 = **495**.

**The Problem** is solved in the MATHCOUNTS Mini.

**Follow-up Problems**

4. Since we are only interested in the order in which Annika (A), Billy (B) and Catherine (C) are called, we can ignore the other 8 students in Ms. McGinn's class. The different orders in which the triplets can be called are ABC, ACB, BCA, BAC, CAB and CBA. In two of these cases, BCA and BAC, Billy is called first. Therefore, the probability that Billy is the first triplet called to Ms. McGinn's desk is 2/6 = **1/3**.

Another way to look at it is the likelihood of being the first triplet called is the same for each triplet. Therefore, the probability of Billy being called first is **1/3**.

5. If the 8-digit number formed by arranging the numbers 1 through 8 is to be a multiple of 5 the units digit must be 5. Once the 5 has been placed, there are seven digits from which to choose the ten-millions digit. Choosing the 6, 7 or 8 will result in a number greater than sixty million.Since 3 of the 7 choices satisfy the condition, we have a probability of **3/7**.

You could also consider that once the 5 has been placed, the digits 1 through 4 and 6 through 8 must be arranged in the remaining seven places. There are a total of $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7! = 5040$ such 8-digit numbers. Only the 8-digit numbers with a millions digit of 6, 7 or 8 are greater than sixty million. There are $1 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6! = 720$ of these 8-digit numbers with a millions digit of 6, 720 with a millions digit of 7 and another 720 with a millions digit of 8. Thus, there are 720 + 720 + 720 = 2160 such 8-digit numbers that are greater than sixty million. It follows that the probability that a number formed by arranging the numbers 1 through 8 is divisible by 5 and greater than sixty million is 2160/5040 = **3/7**.

6. Using the technique employed in the video, we can see that two sums can be formed using each of the corner squares 1, 3, 10 and 12. Figure 1 shows the placements of the domino which include the square containing the number 1. Then three sums can be made with each of the side squares 2, 4, 6, 7, 9 and 11. Figure 2 shows the placements that include the square containing the number 2. Finally, four sums can be made with the two interior squares 5 and 8. Figure 3 shows the placements that include the square containing the number 5. The sum of all these values is 2(1 + 3 + 10 + 12) + 3(2 + 4 + 6 + 7 + 9 + 11) + 4(5 + 8) = 2(26) + 3(39) + 4(13) = 52 + 117 + 52 = **221**.

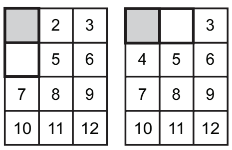
Figure 1

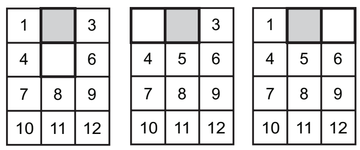
Figure 2

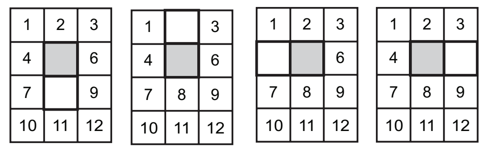
Figure 3

7. Again, using the same technique from the video, we see that each of the corner squares 1, 3, 10 and 12 can be used to make two different sums. Figure 4 shows the placements of the tetromino that include the square containing the number 1. The squares 2 and 11 are each part of four different sums. Figure 5 shows the placements that include the square containing the number 2. Each of the side squares 4, 6, 7 and 9 are part of five different sums. Figure 6 shows the placements that include the square containing the number 4. The two center squares 5 and 8 are each included in eight different sums. Figure 7 shows the placements that include the square containing the number 5. The sum of all these values is 2(1 + 3 + 10 + 12) + 4(2 + 11) + 5(4 + 6 + 7 + 9) + 8(5 + 8) = 2(26) + 4(13) + 5(26) + 8(13) = 52 + 52 + 130 + 104 = **338**.

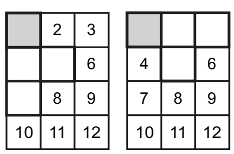
Figure 4

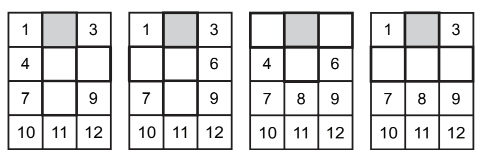
Figure 5

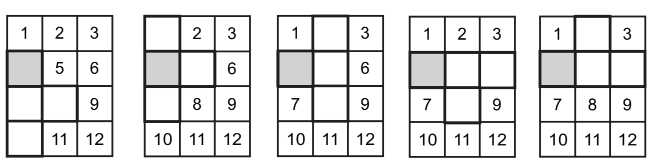
Figure 6

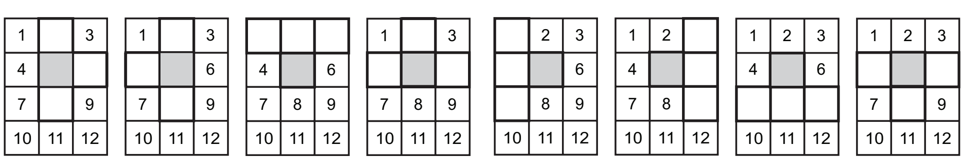
Figure 7

8. The 3-digit palindromes are of the form 1_1, 2_2, 3_3, 4_4, 5_5, 6_6, 7_7, 8_8 and 9_9, where each blank can be any integer from 0 to 9. That means there are 10 palindromes of the form 1 _1, 10 of the form 2_2, and so forth. It follows that there are 90 3-digit palindromes. We can take a look at the sums of the digits in each of the units, tens and hundreds places. In this group of 90 palindromes there are 10 with a units digit of 1, 10 with a units digit of 2, ten with a units digit of 3, and so forth. The sum of these units digits then is 1(10)(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) = 10(45) = 450. Similarly, there are 10 palindromes with a hundreds digit of 1, 10 with a hundreds digit of 2, ten with a hundreds digit of there, and so forth. The sum of the numbers represented by these hundreds digits is 100(10)(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) = 1000(45) = 45,000. Lastly, since the integers from 0 to 9 appear in the tens place for each of the nine palindrome forms previously listed, the sum of the numbers represented by these tens digits is 10(9)(0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) = 90(45) = 4050. Therefore, the sum of all the 3-digit palindromes is 45,000 + 4050 + 450 = **49,500**.