Activity Sheet for the January, 2012, MATHCOUNTS Mini

Warm-Up!

Try these problems before watching the lesson.

1. If a and b are integers such that $a = b^2 - 3$, then what is the smallest possible value of a?

2. Expand each of the following squares of binomials:

   (a) $(x + 1)^2$.
   (b) $(x + 2)^2$.
   (c) $(x + 3)^2$.

3. What number must we place in the blank below to make the resulting quadratic a square of a binomial:
   $$x^2 + 14x + \_\_?$$

4. Expand the expression $(a + b + c)^2$.

The Problem

First Problem: Given the funciton $y = x^2 + 10x + 21$, what is the least possible value of y?

Second Problem: Suppose x and y are real numbers such that $xy = 9$ and

$$x^2 y + xy^2 + x + y = 100.$$

What is the integer value of $x^2 + y^2$?

Third Problem: If $x + y + z = 7$ and $x^2 + y^2 + z^2 = 19$, then what is the arithmetic mean of the three products xy, yz, and xz?

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Follow-up Problems

5. If $x + y = 3$ and $x^2 + y^2 = 6$, then what is xy?

6. Compute $20122011^2 - 2(20122011)(20122009) + 20122009^2$.

7. Suppose that $a + \frac{1}{a} = 6$. What is $a^4 + \frac{1}{a^4}$?

8. If $xyz = 45$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{5}$, then what is the arithmetic mean of the three products xy, yz, and zx?

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).