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Activity Sheet for the April, 2011, MATHCOUNTS Mini

Warm-Up!

Try these problems before watching the lesson. As background, each of the following are arithmetic sequences:

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10$$

$$38, 34, 30, 26, 22, 18, 14, 10, 6, 2, -2$$

$$\frac{1}{2}, \frac{5}{6}, \frac{7}{6}, \frac{3}{2}, \frac{11}{6}, \frac{13}{6}, \frac{5}{2}$$

1. What is the middle term in the arithmetic sequence that consists of the first 31 positive integers?

2. The arithmetic sequence

$$3, 7, 11, \ldots, 83$$

   has 21 terms. What is the middle term?

3. An arithmetic sequence with 14 terms has 31 and 33 as its middle two terms. What is the largest term of the sequence?

4. What is the mean of the first 50 positive even integers?

5. If you write the first 50 positive even integers in order, what are the middle two numbers?

6. Peter, James, Jenny, and Ulana are on a MATHCOUNTS team together. Unfortunately, Peter and James stayed up all night the night before the State contest, and their scores on the Sprint Round were 7 and 8, respectively. Jenny and Ulana got plenty of sleep, and their scores were 21 and 24, respectively.

   (a) What is the average score of the team members on the Sprint Round?

   (b) By how much total were the boys' scores below the team average?

   (c) By how much total were the girls' scores above the team average?

The Problem

First Problem: The sum of the first 20 positive even integers is also the sum of four consecutive even integers. What is the largest of these four integers?

Second Problem: The measures of the interior angles of a convex hexagon form an increasing arithmetic sequence. How many such sequences are possible if the hexagon is not equiangular and all the angle degree measures are positive integers less than 150 degrees?

Third Problem: A set of seven distinct positive integers has its mean and its median both equal to 30. What is the largest possible integer this set can contain?

Follow-up Problems

7. The sum of seven consecutive odd integers is 273. What is the largest of the integers?

8. The sum of the first five terms of an arithmetic sequence is 75. The sum of the first eleven terms of this sequence is 363. What is the first term of the sequence?

9. The sum of the first 9 consecutive odd integers is 3 more than the sum of 6 consecutive even integers. What is the largest of the even integers?

10. When 210 is written as the sum of the greatest possible number of consecutive *positive* integers, what is the largest of these integers? *(Source: MATHCOUNTS)*

11. The degree measures of the interior angles of a convex octagon form an increasing arithmetic sequence of positive integers. How many such sequences are possible?

12. Your average on your first 6 bowling games is 183. What must you score on the seventh game to raise your average to 190? (Try to do this problem in your head!)

13. For a set of 10 numbers, removing the largest number decreases the average by 1. Removing the smallest number increases the average by 2. What is the positive difference between the largest and the smallest of these ten numbers?

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).