MATHCOUNTS Minis February 2011 Activity Solutions

Note: The desired form of the probabilities is not stated in many of the problems, so equivalent percents, fractions or decimals would be considered correct for those problems.

Warm-Up!

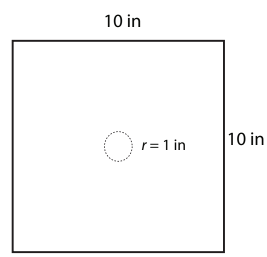
1. Buzz can choose any of the integers 1, 2, 3, 4, 5, 6, 7, 8, 9 or 10. The integers 6, 7, 8, 9 and 10 are all closer to 10 than they are to 1. Therefore the probability that he will choose a number that is closer to 10 than it is to 1 is 5/10 = **1/2**.

2. Tina can choose any of the integers 1, 2, 3, 4, 5, 6, 7, 8 or 9. The integers 6, 7, 8 and 9 are all closer to 9 than they are to 1. Thus the probability that she will choose a number that is closer to 9 than it is to 1 is **4/9**.

3. Since half of the numbers between 0 and 1 are closer to 0 and the other half are closer to 1, we see that the probability of Ari choosing a number that is closer to 1 than it is to 0 is **1/2**.

4. We know that the numbers between 0 and 0.5 are closer to 0.5 than they are to 0.8 and that the numbers between 0.8 and 1 are closer to 0.8. Now consider the numbers between 0.5 and 0.8. Half of those numbers are closer to 0.8 than they are to 0.5. Since the point midway between 0.5 and 0.8 is 0.65 it follows that any number between 0.65 and 1 is closer to 0.8 than it is to 0.5. Since 1 − 0.65 = 0.35 we see those numbers account for 35/100 = 7/20 of the numbers between 0 and 1. Therefore, the probability of Polly choosing a number that is closer to 0.8 than it is to 0.5 is (7/20)/1 = **7/20**.

5. Points that are within 1 inch of the center of the square are in the circular region in the center of the square with a radius of 1 inch. The area of this region is π in^2. The area of the square is $10 \times 10 = 100$ in^2. Therefore, the probability that a point chosen at random is within 1 inch of the center of the square is $\mathbf{\pi/100 \approx 0.031}$.

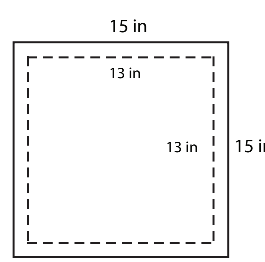


The Problem is solved in the MATHCOUNTS Mini.

Follow-up Problems

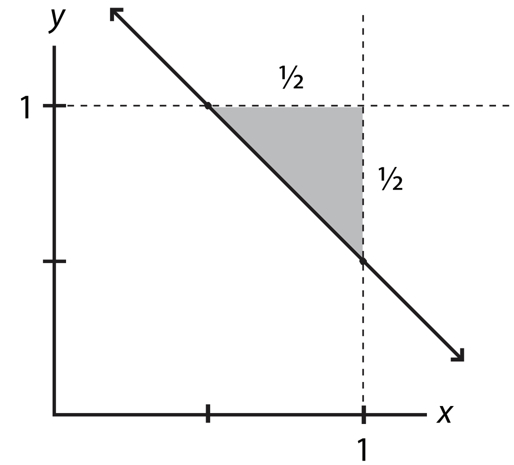
6. Let's think of the stick as a number line of length 1 unit and let the point at which Shannon breaks the stick be x. The distance from 0 to point x is x. We are interested in the case when that distance is more than twice the distance from x to 1, or when the distance from x to 1 is more than twice the distance from 0 to x. Thus, $x \geq 2(1 - x)$ or $1 - x \geq 2x$. Solving for x we have that $x \geq 2/3$ or $x \leq 1/3$. Thus, Shannon is successful if the break point occurs within the first third of the stick or the last third. In other words, the probability that Shannon will break the stick such that the longer piece is more than twice the length of the shorter piece is 1/3 + 1/3 = **2/3**.

7. If the coin is to be entirely on the table Larry's point cannot be within 1 inch of the sides of the table. That means he must choose a point that is located within a 13 by 13 square region in the center of the table. The area of this region is $13 \times 13 = 169$ in^2 and the area of the entire table is $15 \times 15 = 225$ in^2.
So the probability of Larry choosing a point such that the coin is entirely on the table is **169/225**.



8. We are interested in pairs of numbers, *x* and *y*, such that their sum is greater than 3/2. Algebraically, we have $x + y > 3/2$. Using the technique from the video we graph $x + y = 3/2$.

The shaded region represents the set of all pairs, *x* and *y*, such that their sum is greater than 3/2. The area of the shaded region is 1/2(1/2)(1/2) = 1/8 and the area of the entire region is 1(1) = 1. Thus the probability of selecting an *x* and *y* pair that fits the stated criteria is (1/8)/1 = **1/8**.



9. Let *W* represent the number of minutes after 10:00 a.m. when Richard's wife arrives at the airport and let *R* represent the number of minutes after 10:00 a.m. when Richard arrives at the airport. We are interested in pairs of numbers, *W* and *R*, when the two will not be at the airport during the same time, but it is easier to conceptualize when they *are* at the airport together. First, let's think about when Richard's wife arrives, *W*. If she arrives after Richard, but before 30 minutes after Richard's arrival, *R*, they will meet. That is when $W \le R + 30$. But there is a constraint on Richard's arrival time, as well, to account for when his wife arrives at the airport prior to his arrival. In this case Richard's arrival time, *R*, must be before 10 minutes after his wife's arrival, *W*. That is when $R \le W + 10$. Again, using the technique described in the video we graph the lines $W = R + 30$ and $R = W + 10$.

We are interested in the region between these two lines which represents *W* and *R* pairs when Richard and his wife will miss each other at the airport. Instead of trying to determine the area of this irregular region, it is easier to determine the areas of the two triangles and subtract those areas from the area of the total 60 by 60 region. The area of the upper triangle is (1/2)(30)(30) = 450. The area of the lower triangle is (1/2)(50)(50) = 1,250. The area of the entire region is (60)(60) = 3,600. So, the area of the shaded region is 3,600 − 450 − 1250 = 1,900. Therefore, the probability that Richard will be in trouble is 1,900/3,600 = **19/36**.