

MATHCOUNTS[®] Problem of the Week Archive

One for the Ages – January 22, 2024

Problems & Solutions

Cara was born on January 1, 2010, and her mother, Sydney, was born on January 1, 1982. In what year will Sydney's age be twice Cara's age?

If we let Cara's age be C , then Sydney's age is $S = C + 28$, since Cara was born 28 years after Sydney. We are interested in determining when Sydney's age is twice Cara's age, in other words, when $S = 2C$. Substituting, we have $C + 28 = 2C$. Solving for C we get $C = 28$. Therefore, Sydney's age will be twice Cara's age in the year $2010 + 28 = \mathbf{2038}$.

In 2018, Cara's brother, Nile, celebrated a birthday on January 4th. Cara's age at that time was $\frac{4}{5}$ Nile's age at that time. How old was Nile's mom, Sydney, when he was born?

Cara's age in 2018 was $2018 - 2010 = 8$ years old. We are told that Cara's age in 2018 was $\frac{4}{5}$ Nile's age in 2018, N . That means $(\frac{4}{5})N = 8$. Solving for N , we see that Nile's age in 2018 was $N = (\frac{5}{4})8 = 10$ years old. Sydney's age in 2018, S , was $2018 - 1982 = 36$ years. Therefore, ten years prior, when Nile was born, Sydney's age was $36 - 10 = \mathbf{26}$ years old.

The sum of the ages of Cara, Nile and Sydney each year forms an arithmetic progression. The sum of their ages in 2018 was 54. In what year will the sum of their ages be 78?

From the previous problem, we know that Cara's, Nile's and Sydney's ages in 2018 were 8, 10 and 36, respectively. The sum of these ages is $8 + 10 + 36 = 54$. The following year, the sum of their ages was $9 + 11 + 37 = 57$. The following year, the sum of their ages was $10 + 12 + 38 = 60$. Notice that the common difference in the arithmetic progression is $+3$, since each year the ages of Cara, Nile and Sydney each increase by 1. To determine in how many years the sum of the ages will be 78, we can find how many times 3 is added to 54 to get to 78. In other words, we can solve the equation $54 + 3x = 78 \rightarrow 3x = 24 \rightarrow x = 8$. So, the sum of the ages of Cara, Nile and Sydney will be 78 in the year $2018 + 8 = \mathbf{2026}$.

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