MATHCOUNTS®

2024 STATE COMPETITION
Countdown Round Problems 1−80

This booklet contains problems to be used in the Countdown Round.
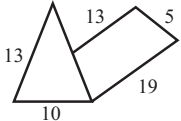
1. _____ Three fair six-sided dice are rolled. What is the probability that the product of the numbers showing on the dice is divisible by 5? Express your answer as a common fraction.

2. _____ If the ratio of the side lengths of two squares is 1:2, what is the ratio of the area of the smaller square to the larger square? Express your answer as a common fraction.

3. _____ (boxes) The *You'll Walk All Over Us* tile store sells 4-inch by 4-inch ceramic tiles in boxes of 40. Brent purchased the minimum number of 4-inch by 4-inch ceramic tiles needed to cover a 10-ft by 12-ft room. How many boxes did he buy?

4. _____ (years old) Ten years from now, Theo will be twice as old as he was 4 years ago. How old is Theo now?

5. _____ What is the value of $\sqrt{7^2 + 24^2 + 36^2 + 48^2}$?

6. _____ What positive integer n satisfies $\sqrt{n + \sqrt{2n}} = 12$?

7. _____ (sides) A convex polygon has an unknown number of sides. When all the diagonals from a single vertex are drawn, seven non-overlapping triangles are created. How many sides does the polygon have?

8. _____ (pencil packs) Jayell had $25 saved up to buy school supplies to donate. Notebooks cost $3.99, pencil packs cost $1.99, and pen packs cost $2.99. Jayell wants to buy at least one of each item. What is the greatest number of pencil packs she can buy?

9. _____ (minutes) The local cooking school offers five classes each Friday. Each class lasts 1 hour 15 minutes. There is a 20-minute break between classes. If the first class begins at 8:00 a.m., how many minutes past noon is it when the final class finishes?

10. _____ (pounds) Four pumpkins have an average weight of 15 pounds, and the average weight of three of those pumpkins is 13 pounds. How many pounds does the fourth pumpkin weigh?

11. _____ There are two numbers twice as far from 15 as they are from 75. What is the sum of those two numbers?

12. _____ A *triangular number* is a positive integer that can be written in the form $\frac{n(n+1)}{2}$, where n is an integer. In 1796, Carl Friedrich Gauss proved that every positive integer is the sum of at most three triangular numbers. What is the greatest of the 3 distinct triangular numbers that sum to 40?

13. _____ For what value of x do the points $(x, x - 2)$, $(x - 3, x)$ and $(0, 0)$ lie on the same line? Express your answer as a common fraction.

14. _____ What is the positive value of x that satisfies $\sqrt{\sqrt{x} + 600} = 25$?

15. _____ Ethan tosses a fair coin eight times. What is the probability that the number of times the coin lands heads up is a multiple of three? Express your answer as a common fraction.

16. _____ (dollars) Gabriel and his two brothers went to the aquarium and spent a total of $116. Gabriel spent $9 more than Mario, and Ernesto spent $4 less than Mario. How many dollars did Ernesto spend?

17. _____ (miles) Virginia rides her bike every day. She rides 12 miles every Monday, 5 miles every Tuesday, 15 miles every Saturday, and 6 miles every other day of the week. Given that May 1, 2023 was a Monday, how many miles did Virginia ride in May of 2023?

18. _____ (in²) A rectangular cake is sliced lengthwise eight times and widthwise six times to form 1-inch by 1-inch squares on the top surface. In square inches, what is the area of the top surface of the cake?

19. _____ What is the greatest negative real number k for which $x^2 + kx - 2k = 0$ for some real number x?

20. _____ (lines) How many lines of symmetry does a regular 18-gon have?

21. _____ (pink Martians) Davig met some space Martians on the way to school. The pink Martians had five eyes and the green Martians had four eyes. There were three times as many pink Martians as green Martians. If Davig saw 133 Martian eyes, how many pink Martians did he see?

22. _____ What is the value of $\dfrac{5^2 - 1^2 + 6^2 - 2^2 + 7^2 - 3^2}{4}$?

23. _____ What number must be subtracted from 1700 to equal the absolute difference between 2564 and 875?

24. _____ (cornstalks) Mrs. Green can buy six pumpkins for the same price as 15 cornstalks. How many cornstalks can she buy for the price of eight pumpkins?

25. _____ (ways) In how many ways can Alice, Bob and four other friends stand in a line if Alice cannot stand next to Bob?

26. _____ Let $a \text{❖} b = |a - b| + |b - a|$. What is the value of $9 \text{❖} 7$?

27. _____ (integers) How many two-digit integers are multiples of eight?

28. _____ (posts)   On a blog, Mackenzie writes a post every three days and Carly writes a post every five days. On days that they are both scheduled to post, they instead collaborate on a longer post. Given that Mackenzie and Carly wrote a collaborative post on January 1, 2023, how many collaborative posts did they write during the entire 2023 calendar year?

29. _____ (degrees)   For a given scalene triangle, two vertices are selected at random. What is the expected value of the sum of the degree measures of the interior angles at the two selected vertices?

30. _____   The value of $\left(1-\frac{1}{2}\right) \times \left(1-\frac{2}{3}\right) \times \left(1-\frac{3}{4}\right) \times \cdots \times \left(1-\frac{9}{10}\right)$ can be expressed as $\frac{1}{n!}$. What is the value of n?

31. _____ (elements)   If A, B and C are non-identical sets that each have 18 elements, what is the least possible number of elements in the union of A, B and C?

32. _____   What is the greatest prime factor of 333×17?

33. _____ (%)   Ronald bought a new watch for the discounted price of $28.60. If the original price of the watch was $44.00, what percentage of the original price did Ronald pay? Express your answer to the nearest whole percent.

34. _____ (degrees)   What is the average degree measure of the interior angles of a 10-sided convex polygon?

35. _____   Boris rolls two fair six-sided dice. What is the expected value of the product of the two numbers he rolled? Express your answer as a common fraction.

36. _____   What is the value of $\dfrac{\text{LCM}(23, 24)}{\text{GCD}(23, 24)}$?

37. _____ (years)   Randy is the youngest of four brothers whose ages are all distinct positive integers. The average of the four brothers' ages is 10 years. What is Randy's greatest possible age, in years?

38. _____   What is the value of $\dfrac{20^{20}}{10^{10} \times 8^8 \times 5^5}$?

39. _____   What is the units digit of $2023^{2021} + 2021^{2023}$?

40. _____ (prime numbers)   How many prime numbers p exist for which $p < 100$ and $p + 1$ is a perfect square?

41. _____ (chicken wings)   A restaurant sells chicken wings in baskets of 5 or 8. What is the greatest exact number of chicken wings that cannot be purchased with whole baskets?

42. _____ (perfect cubes)   How many positive perfect cubes have four digits?

43. _____ (inches)   The distance between the centers of two circles is 42 inches, and the distance between the two circles is 27 inches. What is the sum of the lengths of the radii of the circles, in inches?

44. _____   Two different positive integers have a product equal to seven times their sum. What is the greater of the two integers?

45. _____ (ft²)   What is the surface area, in square feet, of a cylinder with diameter 1 foot and height 1 foot? Express your answer as a common fraction in terms of π.

46. _____   The arithmetic mean of 10, 9, 3, x and y is 5. What is the value of $x + y$?

47. _____   Two numbers have a sum of 36. What is the greatest possible product of the two numbers?

48. _____   What value of t satisfies the equation $24t - 20 = 8$? Express your answer as a decimal to the nearest hundredth.

49. _____ (minutes)   Justin, Pierce and Eli are running a race. Justin runs twice as fast as Eli, and Pierce runs three times as fast as Eli. If Justin finishes the race in 36 minutes, how many minutes does it take Pierce to finish the race?

50. _____   What is the units digit of $3^{2024} - 2^{2023}$?

51. _____   An unfair coin is flipped twice. The probability that both flips land heads up is greater than zero and is the same as the probability that one flip lands heads up and the other lands tails up. What is the probability that both flips land heads up? Express your answer as a common fraction.

52. _____ (in²)   An isosceles triangle with two sides of length 13 inches and a base of length 10 inches is attached to an isosceles trapezoid with a slant height of 5 inches and bases of lengths 13 and 19 inches, as shown. What is the area of the resulting figure, in square inches?

53. _____   A triangle has side lengths $\sqrt{101}$, $\sqrt{102}$ and n, where n is an integer. What is the greatest possible value of n?

54. _____ (perfect squares)   How many positive perfect squares have fewer than four digits?

55. _____   If the product $\sqrt{15} \times \sqrt{25} \times \sqrt{35} \times \sqrt{n}$ is an integer, what is the least possible positive integer value of n?

56. _____ (meters)   Fran builds a fence around her square-shaped backyard to keep her new puppy safe. If the area of her backyard is 1024 m², what is the minimum length of fencing, in meters, that Fran will need to enclose her backyard?

57. _____ (ounces)   Oscar has 4 adult cats and 14 kittens. The adult cats each eat 3.5 ounces of food twice a day, while the kittens each eat 1.2 ounces of food three times a day. How many ounces of cat food do Oscar's cats eat in one week? Express your answer as a decimal to the nearest tenth.

58. _____ (integers)   How many positive integers less than 320 are relatively prime to 155?

59. _____   If x and y are positive numbers, their root mean square is defined as $\sqrt{\dfrac{x^2 + y^2}{2}}$. What is the root mean square of 2 and 14?

60. _____   What is the absolute difference between the mean and the median of the prime numbers less than 20? Express your answer as a common fraction.

61. _____ (days)   If three carpenters can build three fences in three days, then how many days does it take one carpenter to build two fences?

62. _____   If $0.45 = \dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{n}$, what is the value of n?

63. _____   If n is a two-digit integer for which $n + 1$, $\dfrac{n}{2} + 1$, $\dfrac{n}{3}$ and $\dfrac{n}{6} + 1$ are all perfect squares, what is the value of n?

64. _____   The distance between the points $(2, -1, 9)$ and $(-1, x, 21)$ is 13. What is the sum of all possible values of x?

65. _____   What is the least positive integer r for which 2025 is a divisor of $r!$?

66. _____   If $f(x) = x + 3$ and $g(x) = 3x^2 - 2x + 7$, what is the value of $g(f(5))$?

67. _____ (%)   A rectangle has a width of 17 cm and a height of 23 cm. A new rectangle is constructed so that both the width and height are 50% greater than the width and height, respectively, of the original rectangle. The area of the new rectangle is what percent of the area of the original rectangle?

68. _____ (marbles)   A jar contains only yellow and green marbles, and the ratio of yellow to green marbles is 4:7. After Nolan adds 50 yellow marbles to the jar, the ratio of yellow to green marbles is 2:1. How many marbles were in the jar originally?

69. _____   What is the greatest integer value of n for which 10^n is a divisor of $(6!)!$?

70. _____   The sum of the digits of m plus the product of the digits of m is equal to 26. If m is a two-digit number, what is the greatest possible value of m?

71. _____ (dollars)   A shop sells all teacups at one price and all saucers at another price. Carson pays $67 for three teacups and two saucers. Daylon pays $57 for a teacup and five saucers. How many dollars will Eidan pay for one teacup?

72. _____ (cupcakes)   Devlyn made a batch of between 50 and 70 cupcakes. When they are packaged in groups of 3, 4 or 5 cupcakes, there is always one cupcake left over. How many cupcakes will be left over if they are packaged in groups of 7 cupcakes?

73. _____   The average of six numbers is 84. If 2 is added to the first number, 4 is added to the second number, 6 is added to the third number and so on, until 12 is added to the sixth number, what is the new average of these six new numbers?

74. _____ (dollars)   Mr. Florimonte spent $40 for two books. One book cost $8 more than the other book. How much did Mr. Florimonte spend for the less expensive book?

75. _____ (arrange-ments)   How many distinct arrangements are there of the letters in the word ENGINEER?

76. _____ (degrees)   In regular 10-gon MATHCOURSE, shown here, what is the degree measure of angle HOT?

77. _____   What is the greatest integer d for which $\dfrac{(d+1)^2}{16d} < 1$?

78. _____   After expansion, what is the sum of the coefficients in $(x + 6)^2$?

79. _____ (units^2)   A triangle is bounded by the lines $y = x + 4$, $y = -x + 4$ and $y = 0$. What is the area of the triangle, in square units?

80. _____   Chip divides the number a by 7 and gets 5.25 as his answer. What is the result when Dale multiplies a by 8?