MATHCOUNTS® Problem of the Week Archive

Neuroscience Research – March 25, 2024

Problems & Solutions

The following information and problems were submitted by a MATHCOUNTS volunteer, Emily Koithan. Thank you, Emily!

Neuroscientists use a special type of brain scanning called functional magnetic resonance imaging, or fMRI, to answer questions about how our brains work. It can tell us what areas of the brain are active when we move, listen and think.

A researcher asks a study participant to tap their finger during an fMRI scan and observes activity in 10 different brain regions. If the activity in each region is compared to the activity in every other region, how many comparisons are made?

Let's say the 10 brain regions are labeled with the numbers 1 through 10. We can methodically count the comparisons made by starting with region 1. Region 1 is separately compared to regions 2, 3, 4, 5, 6, 7, 8, 9 and 10, which is a total of 9 comparisons. Moving on to region 2, we see that the comparison between region 1 and region 2 has already taken place. Thus, region 2 still needs to be compared to regions 3, 4, 5, 6, 7, 8, 9 and 10, which is a total of 8 comparisons. At this point, region 3 has already been compared to regions 1 and 2, so region 3 still needs to be compared to regions 4, 5, 6, 7, 8, 9 and 10. This is a total of 7 comparisons. We can continue with this pattern to find that if the activity in each brain region is compared to the activity in every other brain region, there will be 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = ***45*** *comparisons.*

Alternatively, each of the 10 brain regions will be compared with 9 other brain regions (every region but itself). However, if we simply multiply 10 × 9, we will have double-counted each of the comparisons. For example, we will have counted when region 1 is compared to region 2 and when region 2 is compared to region 1. This is actually the same comparison, though, since each brain region is only sharing one comparison with each other region. Thus, we'll need to account for this by dividing by 2, so there will be (10 × 9)/2 = ***45*** *comparisons.*

Alternatively, we can use the combinations formula, which says that the total number of combinations (in this case, comparisons) is n!/[r!(n – r)!], where n = the total number of brain regions and r = the number of brain regions to select for each comparison. Since there are 10 brain regions and 2 regions in a single comparison, using the combinations formula, we find that there will be 10!/[2!(10 – 2)!] = 10!/[2!8!] = (10 × 9)/(2 × 1) = 90/2 = ***45*** *comparisons.*

The researcher then plays music and looks at activity in the participant's auditory cortex, one part of the brain active when you hear sounds. She plays the music louder, and the activity in the auditory cortex increases by 60%. Then, she plays the music softer, and the activity in the auditory cortex decreases by 14%. If *x* represents the original activity level, by what percent does the activity in the participant's auditory cortex increase while the music is playing? Express your answer as a decimal to the nearest tenth.

Let the original activity in response to the music be given by x. Then, when the music is played louder, the increase in activity of 60% can be represented by 1.6x. When the music is lowered and the activity decreases by 14%, this can be represented by 0.14(1.6x). Thus, the activity at the end is 1.6x – 0.14(1.6x) = 1.6x – 0.224x = 1.376x. So, the activity is 1.376 times higher at the end than it was when the music originally started to play, which is an increase of ***37.6****%.*

Researchers often study the brain using voxels, which are tiny three-dimensional cubes. The researcher finds that in one participant, the inferior parietal lobule (a region of the brain active when performing arithmetic) has a volume of 22 cubic centimeters. Suppose the inferior parietal lobule in that participant contains 2,750 voxels. How long is the side of one voxel in *millimeters*?

The volume of one voxel is given by 22 cm^3 / 2750 = 22,000 mm^3 / 2750 = 8 mm^3. Since the volume of a cube is given by length × width × height, the side of each voxel is **2** *millimeters long.*

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