

MATHCOUNTS[®] Problem of the Week Archive

Math Club Meetings – September 4, 2023

Problems & Solutions

Ms. Cross leads The National Math Club at the middle school where she teaches. At the first club meeting of the school year, 60% of the students in attendance were 7th graders. If there was one fewer 8th grader than 7th grader in attendance, how many students attended the first club meeting?

Let a and b represent the number of 7th graders and 8th graders who attended the first club meeting, respectively. We know that $a = b + 1$ and $a = 0.6(a + b)$. Solving for a in the second equation yields $a = 0.6a + 0.6b \rightarrow 0.4a = 0.6b \rightarrow a = 1.5b$. Substituting this value for a in the first equation, we get $1.5b = b + 1 \rightarrow 0.5b = 1 \rightarrow b = 2$. So, there were 2 8th graders and $2 + 1 = 3$ 7th graders in attendance at the first club meeting. Therefore, a total of $2 + 3 = 5$ students attended the first club meeting.

At the second club meeting of the school year, Ms. Cross noticed that among the students in attendance, there were equal numbers of 7th graders and 8th graders. Ms. Cross also noticed that all the students from the first club meeting were in attendance, along with some new students who weren't at the first club meeting. If twice as many new 8th graders attended the second club meeting as new 7th graders, how many new students attended the second club meeting?

From the previous problem, we know that there were 3 7th graders and 2 8th graders in attendance at the first club meeting. It follows, then, from the information given, that if x new 7th graders attended the second club meeting, $2x$ new 8th graders attended that meeting. Since the same number of 8th graders and 7th graders attended the second meeting, we can write $3 + x = 2 + 2x$ and solve to get $x = 1$. That means 1 new 7th grader and $2 \times 1 = 2$ new 8th graders attended the second meeting. Therefore, a total of $1 + 2 = 3$ new students attended the second meeting.

At the third meeting of The National Math Club, Ms. Cross noticed that 60% of the students in attendance were 8th graders and that the total number of students in attendance was double that of the first club meeting. Ms. Cross was pleased to see that all the students who attended the second club meeting also attended the third club meeting. Of the students in attendance at the third club meeting who did not attend the first two club meetings, what is the absolute difference between the number of 8th graders and the number of 7th graders?

From the first problem, we know that a total of 5 students attended the first club meeting. Based on the information provided, twice that number, or $5 \times 2 = 10$ students attended the third club meeting. Since 8th graders accounted for 60% of these 10 students, it follows that $0.6 \times 10 = 6$ 8th graders and $10 - 6 = 4$ 7th graders attended the third club meeting. Since 4 7th graders and 4 8th graders attended the second club meeting, there were $4 - 4 = 0$ new 7th graders and $6 - 4 = 2$ new 8th graders at the third club meeting. The absolute difference between these two quantities is $2 - 0 = 2$.

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