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2023 STATE COMPETITION Sprint Round Problems 1–30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

Chapter _____

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DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

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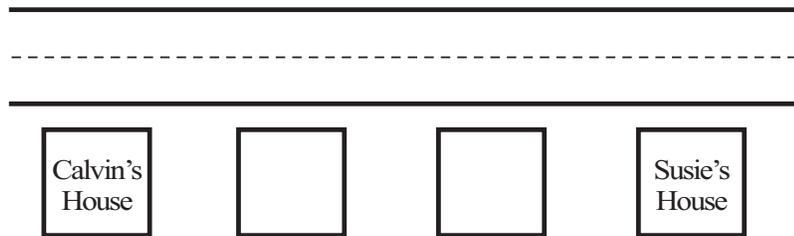
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1. _____ What integer is closest to $\frac{4.7}{2}$?
2. _____ What is the value of $\frac{10^3}{3^4 - 1}$? Express your answer as a decimal to the nearest tenth.
3. _____ What is the greatest integer value of t for which $3t < 260$?
4. _____ degrees In triangle ABC, $m\angle A = 40$ degrees and $m\angle B = 85$ degrees. What is the measure of $\angle C$?
5. _____ The arithmetic mean of 4, 8, 6, 9 and n is 6. What is the value of n ?

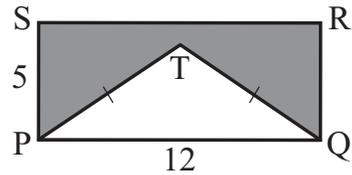
6. _____ Calvin and Susie live on the same side of a street, as shown, where adjacent houses have address numbers that differ by six. If Calvin's address number is 2021 and is less than Susie's address number, what is Susie's address number?



7. _____ mm^2 The perimeter of a rectangle is 36 mm. What is the greatest possible area of the rectangle?
8. _____ cups It requires 5 tablespoons of flour to make 15 gulab jamun. Given that there are 16 tablespoons in a cup, how many cups of flour are required to make 200 gulab jamun? Express your answer as a mixed number.
9. _____ The absolute difference of two positive numbers is 3, and their product is 40. What is their sum?
10. _____ minutes Nautical dawn is the time in the morning when the center of the sun is 12 degrees below the horizon. Civil dawn is the time in the morning when the center of the sun is 6 degrees below the horizon. Assuming that one day corresponds to one full rotation, how many minutes pass between nautical dawn and civil dawn?

16. _____ cm

In the figure shown, PQRS is a rectangle and PQT is an isosceles triangle. Given that the area of pentagon PTQRS is 36 cm^2 , what is length PT? Express your answer in simplest radical form.

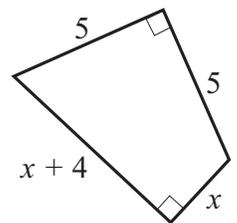


17. _____ meters

Lauren and Ally are going to race. Lauren runs 0.4 m/s faster than Ally. If Lauren is going to run 100 meters, how many meters ahead of Lauren does Ally need to start in order for both runners to reach the finish line in 20 seconds?

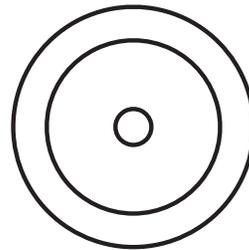
18. _____

The lengths, in units, of the sides of a quadrilateral are given in the figure shown. What is the value of x ? Express your answer in simplest radical form.



19. _____

A dartboard is made up of three concentric circles, as shown, with radii 7 inches, 5 inches and 1 inch. What is the probability that a randomly thrown dart that hits this dartboard will be closer to the circle of radius 5 inches than to either of the other two circles? Express your answer as a common fraction.

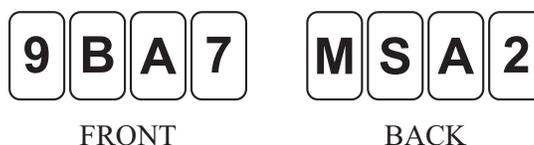


20. _____

The arithmetic mean of two numbers is 7 and their geometric mean is 5. What is the sum of the squares of the two numbers?

21. ordered triples Let $x = 352,ABC,052$, where A, B and C represent digits that are not necessarily distinct. How many ordered triples (A, B, C) exist for which x is divisible by 44?

22. Amira takes four blank cards and writes the letters and numerals shown in the figure on the front of the cards. Amira then flips over the cards, shuffles them, and randomly writes the letters and numerals shown on the back of the cards. What is the probability that the following statement is true for all cards: "If a card has a letter written on one side, then the letter A is not written on the other side of that card."? Express your answer as a common fraction.



23. degrees The line L passes through the point (8, 11), and the smaller angle formed by the line L and the line $y = x$ measures 31 degrees. What is the greatest possible degree measure of the smaller angle formed between the line L and the line $y = x\sqrt{3}$?

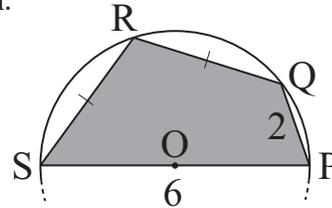
24. entries In the 3×3 array shown, the sums of all 3 rows and all 3 columns are different. Yang wants to change the values of some of the entries in order to make the sum of all 3 rows and all 3 columns the same. For example, one possibility is that Yang could change every entry into the number 9, which would require changing 8 values. What is the fewest possible entries that Yang could change to achieve his goal?

1	7	8
6	3	3
3	5	9

25. What is the greatest two-digit prime factor of $\frac{300!}{200!100!}$?

26. _____ locations Freddy the frog starts at the origin of a coordinate plane. In any given hop, he can hop exactly one unit up, down, left or right. At how many different locations could Freddy possibly land after his 100th hop?

27. _____ cm^2 In the figure shown, quadrilateral PQRS is inscribed in circle O. Given that $QR \cong RS$, $SP = 6$ cm, and $PQ = 2$ cm, what is the area of quadrilateral PQRS? Express your answer in simplest radical form.



28. _____ Suppose x and y are real numbers for which $2xy + 16 = x^2$ and $2xy + 9 = 4y^2$. If $y > 0$, what is the value of $x + y$? Express your answer as a decimal to the nearest tenth.

29. _____ What integer is closest to the value of the expression $\left(\sqrt{3+\sqrt{12}} + \sqrt[4]{1728} + \sqrt{3+\sqrt{12}} - \sqrt[4]{1728}\right)^4$?

30. _____ The volume of a regular dodecahedron of edge length 1 is $\frac{15+7\sqrt{5}}{4}$, and the volume of a regular icosahedron of edge length 1 is $\frac{15+5\sqrt{5}}{12}$. A solid metal icosahedron of edge length 2 is melted down, divided, and used to form two solid shapes: a dodecahedron of edge length $\sqrt[3]{a}$ and a cube of edge length $\sqrt[3]{b}$. If a and b are both rational numbers, what is the value of $a + b$? Express your answer as a common fraction.

