2014–2015
MATHCOUNTS®
School Handbook

Contains 300 creative math problems
that meet NCTM standards for grades 6-8.

For questions about your local MATHCOUNTS program, please contact your chapter (local) coordinator. Coordinator contact information is available through the Find My Coordinator link on www.mathcounts.org/competition.

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Acknowledgments

The MATHCOUNTS Foundation wishes to acknowledge the hard work and dedication of those volunteers instrumental in the development of this handbook: the question writers who develop the questions for the handbook and competitions, the judges who review the competition materials and serve as arbiters at the National Competition and the proofreaders who edit the questions selected for inclusion in the handbook and/or competitions.

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The **Solutions** to the problems were written by Kent Findell, Diamond Middle School, *Lexington, MA.*

MathType software for handbook development was contributed by **Design Science Inc.**, www.dessci.com, *Long Beach, CA.*

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Count Me In!

A contribution to the MATHCOUNTS Foundation will help us continue to make its worthwhile programs available to middle school students nationwide.

The MATHCOUNTS Foundation will use your contribution for programwide support to give thousands of students the opportunity to participate.

To become a supporter of MATHCOUNTS, send your contribution to:

MATHCOUNTS Foundation
1420 King Street
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Or give online at:

www.mathcounts.org/donate

Other ways to give:
- Ask your employer about matching gifts. Your donation could double.
- Remember MATHCOUNTS in your Combined Federal Campaign at work.
- Leave a legacy. Include MATHCOUNTS in your will.

For more information regarding contributions, call 703-299-9006, ext. 107 or email info@mathcounts.org.

The MATHCOUNTS Foundation is a 501(c)3 organization. Your gift is fully tax deductible.

The National Association of Secondary School Principals has placed this program on the NASSP Advisory List of National Contests and Activities for 2014–2015.

CRITICAL 2014-2015 DATES

2014

Aug. 18 - Dec. 12 — Send in your school's Competition Series Registration Form to participate in the Competition Series and to receive the 2014-2015 School Competition Kit, with a hard copy of the *2014-2015 MATHCOUNTS School Handbook*. Kits begin shipping shortly after receipt of your form, and mailings continue every two weeks through December 31.

Mail, e-mail or fax the MATHCOUNTS Competition Series Registration Form with payment to:

> **MATHCOUNTS Registration, P.O. Box 441, Annapolis Junction, MD 20701**
> **E-mail: reg@mathcounts.org**
> **Fax: 240-396-5602**

Questions? Call 301-498-6141 or confirm your registration via www.mathcounts.org/competitionschools. (Please allow 10 days before confirming your registration online.)

Nov. 3 — The 2015 School Competition will be available online. With a username and a password, a registered coach can download the competition from www.mathcounts.org/competitioncoaches.

Nov. 14 — **Deadline to register for the Competition Series *at reduced registration rates*** ($90 for a team and $25 for each individual). After November 14, registration rates will be $100 for a team and $30 for each individual.

Dec. 12 (postmark) — **Competition Series Registration Deadline**
In some circumstances, late registrations might be accepted at the discretion of MATHCOUNTS and the local coordinator. ***Late fees will also apply. Register on time to ensure your students' participation.***

2015

Early Jan. — If you have not been contacted with details about your upcoming competition, call your local or state coordinator!

Late Jan. — If you have not received your School Competition Kit, contact the MATHCOUNTS national office at 703-299-9006.

Jan. 31 - Feb. 28 — **Chapter Competitions**

Mar. 1-31 — **State Competitions**

May 8 — **2015 Raytheon MATHCOUNTS National Competition in Boston, MA**

MATHCOUNTS PROGRAM OVERVIEW

MATHCOUNTS was founded in 1983 as a way to provide new avenues of engagement in math to middle school students. MATHCOUNTS programs help students of all levels reach their full potential—whether they love math or fear it. We help expand students' academic and professional opportunities through three unique, but complementary, programs: the **MATHCOUNTS Competition Series**, **The National Math Club** and the **Math Video Challenge**. This *School Handbook* supports each program in different ways.

MATHCOUNTS® COMPETITION SERIES EST. 1983

The **MATHCOUNTS Competition Series** is a national program that provides bright students the opportunity to compete head-to-head against their peers from other schools, cities and states in four levels of competition: school, chapter (local), state and national. MATHCOUNTS provides preparation and competition materials and, with the leadership of the National Society of Professional Engineers, hosts more than 500 Chapter Competitions, 56 State Competitions and the National Competition each year. This year, the top four students from each U.S. state and territory will compete at the 2015 Raytheon MATHCOUNTS National Competition in Boston, MA. Students win hundreds of thousands of dollars in scholarships each year at the local, state and national levels. There is a registration fee for students to participate in this program and registration is limited only to schools. Participation beyond the school level is limited 10 students per school. More information about the Competition Series can be found on pg. 40-52 of this *School Handbook* and at **www.mathcounts.org/competition**.

Working through the* School Handbook *and past competitions is the best way to prepare for MATHCOUNTS competitions.

THE NATIONAL MATHCLUB powered by MATHCOUNTS®

The National Math Club is a free enrichment program that provides teachers and club leaders with resources to run a math club. The materials provided through The National Math Club are designed to engage students of all ability levels—not just the top students—and are a great supplement for classroom teaching. This program emphasizes collaboration and provides students with an enjoyable, pressure-free atmosphere in which they can learn math at their own pace. Active clubs can also earn rewards by having a minimum number of club members participate (based on school/organization/group size). There is *no cost to sign up* for The National Math Club, and registration is open to schools, organizations and groups that consist of at least four students in 6th, 7th or 8th grade and have regular in-person meetings. More information can be found at **www.mathcounts.org/club**.

The* School Handbook *is supplemental to The National Math Club. Resources in the* Club Activity Book *will be better suited for more collaborative and activity-based club meetings.

MATHVIDEO CHALLENGE produced by MATHCOUNTS®

The **Math Video Challenge** is an innovative program that challenges students to work in teams to create a video explaining the solution to a MATHCOUNTS problem and demonstrating its real-world application. This project-based activity builds math, communication and collaboration skills. Students post their videos to the contest website, where the general public votes for the best videos. The 100 videos with the most votes advance to judging rounds, in which 20 semifinalists and, later, four finalists are selected. This year's finalists will present their videos to the students competing at the 2015 Raytheon MATHCOUNTS National Competition, and those 224 Mathletes will vote to determine the winner. Members of the winning team receive college scholarships. Registration is open to all 6th, 7th and 8th grade students. More information can be found at **videochallengemathcounts.org**.

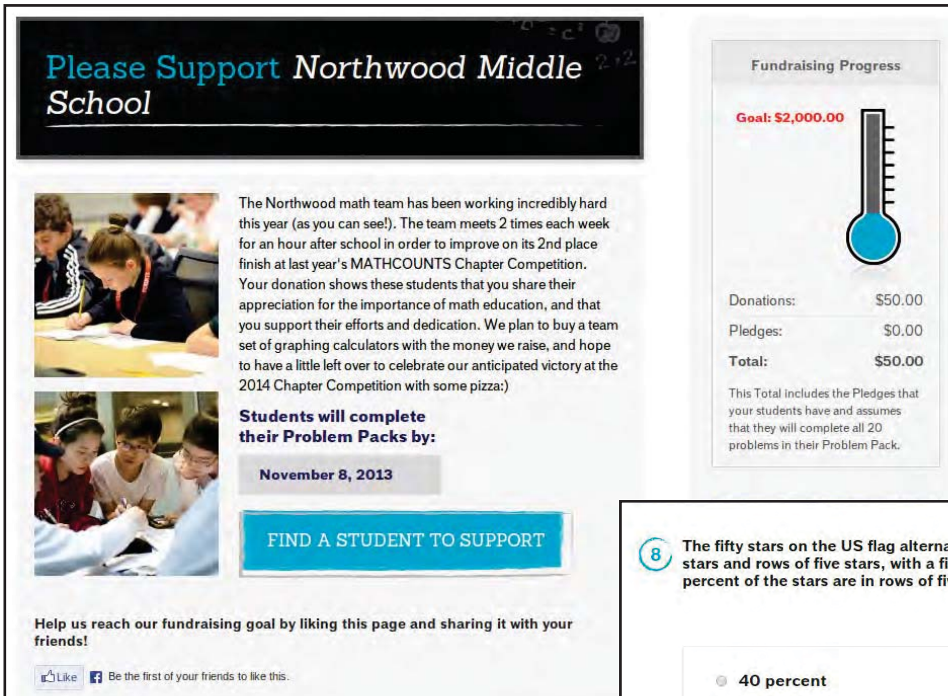
***Students must base their video for the 2014-2015 Math Video Challenge on a problem they select from the* 2014-2015 School Handbook.**

HELPFUL RESOURCES

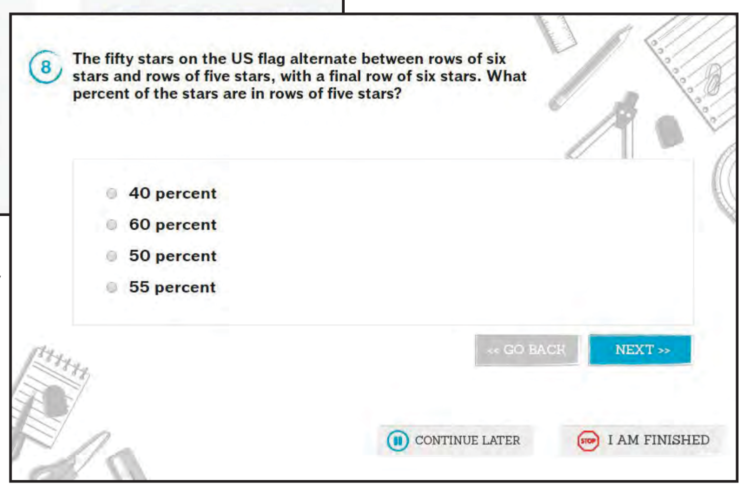
THE MATHCOUNTS SOLVE-A-THON

Solve-A-Thon is a fund-raiser that empowers students and teachers to use math to raise money for the math programs at their school. All money raised goes to support math programming that benefits the students' local communities, with 60% going directly back to the school. Launched last year, Solve-A-Thon was designed with teachers in mind; participating is free, and getting started takes just a couple of minutes. Here's how the fund-raiser works:

1. Teachers and students sign up and **create** online fund-raising pages (example below) that explain why they value math and why they are raising money for their school's math program.



2. Students **share** the links to their fund-raising pages with friends, family and local community members to earn donations and pledges.
3. Students **solve** an online Problem Pack with 20 math problems covering topics from the NCTM Standards for grades 6-8 (example below).
4. Students, teachers, schools and local communities **win** critical funding, prizes and improved math programs.

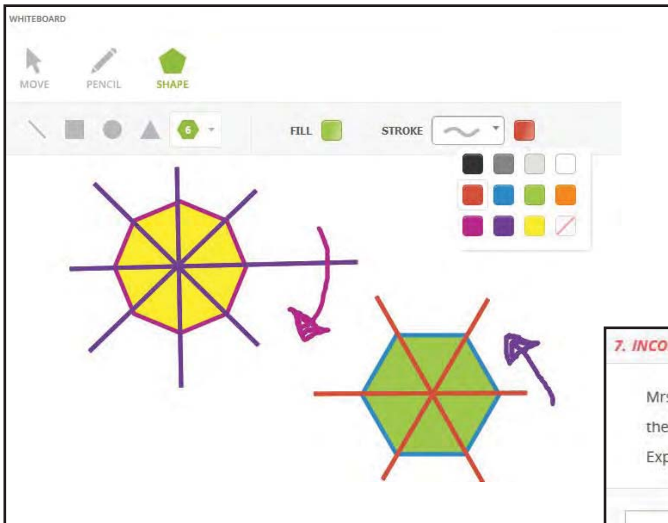


Unlike traditional fund-raisers, there is no door-to-door selling and no tracking of sales or money raised. Students participate in a meaningful learning activity to raise money, and money earned is tracked online automatically. Learn more and sign up at **solveathon.mathcounts.org**.

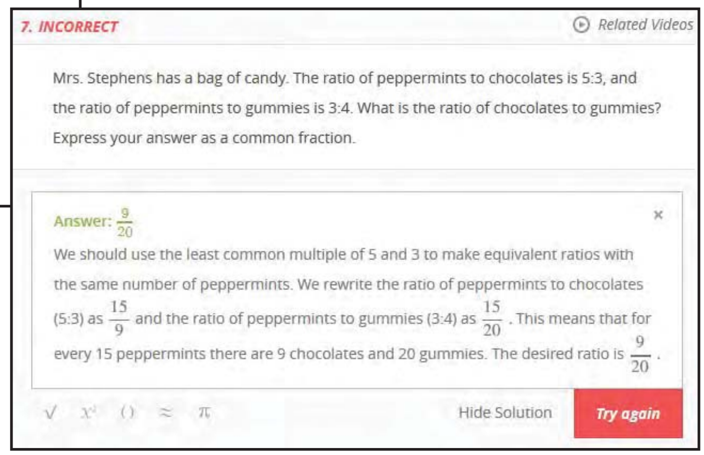
INTERACTIVE MATHCOUNTS PLATFORM

The **Interactive MATHCOUNTS Platform** provides a unique forum where members of the MATHCOUNTS community can collaborate, chat and take advantage of innovative online features as they work on problems from MATHCOUNTS handbooks and School, Chapter and State competitions.

Powered by NextThought, the Interactive MATHCOUNTS Platform continues to grow, with more problems and features being added every year. Currently this resource includes problems from *MATHCOUNTS School Handbooks* from 2011-2012, 2012-2013, 2013-2014 and 2014-2015, as well as School, Chapter and State Competitions from 2012, 2013 and 2014. Users will enjoy a truly engaging and interactive experience. Here are just a few of the features on the platform.



- **Digital whiteboards** (example at left) enable students to highlight problems, add notes and questions and show their work.
- **Interactive problems** (example below) can be used to assess student or team performance.
- **Advanced search tools** make it easy to find MATHCOUNTS content and notes.
- **Collaborative forums** allow users to chat and share with the global MATHCOUNTS community.



See how this incredible tool can add to your MATHCOUNTS experience. Create a free account and try out the Interactive MATHCOUNTS Platform at **mathcounts.nextthought.com**.

THE MATHCOUNTS OPLET

The **Online Problem Library and Extraction Tool** is an online database of over 13,000 problems and over 5,000 step-by-step solutions. OPLET subscribers can create personalized worksheets/quizzes, flash cards and Problems of the Day with problems pulled from the past 14 years of MATHCOUNTS handbooks and competitions.

With OPLET, creating original resources to use with your competition team and in your classroom is easy. You can personalize the materials you create in the following ways.

- **Format**: worksheet/quiz, flash cards or Problem of the Day
- **Range of years** of MATHCOUNTS materials
- **Difficulty level**: five levels from easy to difficult
- **Number of questions**
- **Solutions included/omitted** for select problems
- **MATHCOUNTS usage**: filters by competition rounds or handbook problem types
- **Math concept:** including arithmetic, algebra, geometry, counting/probability, number theory

OPLET Subscriptions can be purchased at **www.mathcounts.org/oplet**. A 12-month subscription costs $275, and schools registering students in the MATHCOUNTS Competition Series receive a $5 discount per registered student (up to $50 off). Plus, if you purchase OPLET by October 17, 2014, you can save an additional $25, for a total savings of up to $75 (see coupon at right).

GET A $25 DISCOUNT!

Save $25 on your OPLET subscription if you purchase or renew by **October 17, 2014**.

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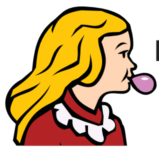
www.mathcounts.org/oplet

Warm-Up 1

1. __________ What is the sum of the two-digit multiples of 11?

2. __________ If $a \# b = a + 2b$, for integers a and b, what is the value of $3 \# 4$?

3. __________ times



Kimba chewed a piece of gum 42 times in one minute. If she continued to chew at the same rate, how many times would she chew her gum in 100 seconds?

4. __________ miles Carver jogs 4 times a week. Last week, his distances were $4\frac{1}{3}$, $3\frac{1}{2}$, $3\frac{5}{6}$ and $4\frac{1}{6}$, all measured in miles. What was Carver's average distance for the 4 days? Express your answer as a mixed number.

5. __________ socks Rob has 10 white, 8 red and 6 blue socks in his drawer. If he selects socks from the drawer randomly, without looking, what is the least number of socks Rob must select to guarantee that he has removed a pair of white socks?

6. __________ hamburgers At a particular restaurant, hamburgers are priced \$3 each, 2 for \$5 and 5 for \$9. What is the maximum number of hamburgers that can be purchased for \$48?

7. Statement __________

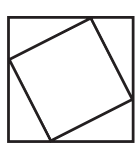
   Statement __________

Which two of the following four statements, labeled A through D, are true statements?

A: Statement B is false, but statement C is true.
B: Statement C is true, but statement D is false.
C: Statement D is false, and statement A is false.
D: Statement A is true, and statement B is true.

8. __________ seconds How many seconds are in 3.14 hours?

9. __________



The vertices of the smaller square in the figure are at trisection points of the sides of the larger square. What is the ratio of the area of the smaller square to the area of the larger square? Express your answer as a common fraction.

10. __________ students Students at Central School were surveyed regarding lunch choices. Of the students that responded, exactly $\frac{1}{3}$ wanted more fresh fruits and vegetables as choices. Of those students not wanting more fresh fruits and vegetables, exactly $\frac{1}{8}$ wanted more seafood. What is the minimum number of students that responded to the survey?

Warm-Up 2

11.__________ Two cylinders are equal in volume. The radius of one is doubled, and the height of the other cylinder is increased to k times its original height. If the two new cylinders are equal in volume, what is the value of k?

12.__________ What positive integer must be included in the set {1, 2, 4, 8} so that the new set of five integers has a median that is equal to its mean?

13.__________ coins

Debbie has equal numbers of dimes and quarters with a total value of \$1.40. How many coins does she have altogether?

14.__________ What is the average of the first 99 counting numbers?

15.__________ minutes How many fewer minutes does it take to drive 35 miles at 30 mi/h than to drive the same distance at 25 mi/h?

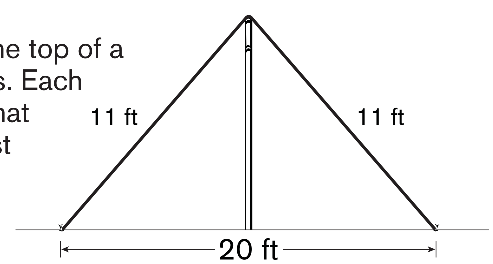
16. \$__________ After a 20% reduction, the price of a computer was \$800. What was the price of the computer before the reduction?

17.__________ ounces Koka-Kola is sold in packages of eight 12-ounce bottles. Pepsy-Kola is sold in packages of six 16-ounce bottles. In ounces, what is the absolute difference in the total number of ounces in a package of Koka-Kola and a package of Pepsy-Kola?

18.__(,)__ Point A lies at the intersection of $y = x$ and $y = -\frac{2}{3}x + 5$. What are the coordinates of A? Express your answer as an ordered pair.

19.__________ When rolling two fair, standard dice, what is the probability that the sum of the numbers rolled is a multiple of 3 or 4? Express your answer as a common fraction.

20.__________ feet Two wires are connected by stakes from the ground to the top of a vertical pole that is anchored halfway between the stakes. Each wire is 11 feet long, and the stakes are 20 feet apart. What is the height of the pole? Express your answer in simplest radical form.

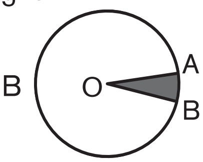


Workout 1

21.__________ What is the sum of the prime factors of 2015?

22.__________ What is $(3.5 \times 10^4)^2$ when written in scientific notation with four significant digits?

23.__________ meters The area of the shaded region of circle O is 9π m^2, and the measure of $\angle AOB$ is 22.5 degrees. What is the length of the radius of circle O?



24. $__________ In Fuelville, the cost of gas averaged \$3.50 per gallon at the start of April, then rose 6% during April and dropped 10% during May. What was the cost of a gallon of gas at the end of May?

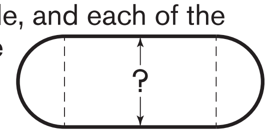
25.__________ ft^2 A square residential lot is measured to be 100 feet on each side, with a possible measurement error of 1% in each of the length and width. What is the absolute difference between the largest and smallest possible measures of the area given this possible error?

26.__________ years old Currently, the sum of the ages of Yumi, Rana and Victoria is 42 years. Four years ago, the sum of the ages of Rana and Victoria was equal to the current age of Yumi. What is Yumi's current age?

27.__________ students

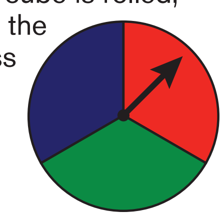
Of 600 students at Goodnight Middle School in Texas, 85% are not native Texans. Of those non-native students, 60% have lived in Texas more than 10 years, and 30 students have lived in Texas less than a year. How many non-native students have lived in Texas for at least 1 year but not more than 10 years?

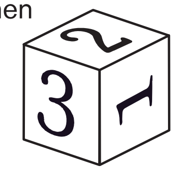
28.__________ miles The track at Dividend Middle School, depicted here, has two semicircular ends joined by two parallel sides. The total distance around the track is 1/4 mile, and each of the semicircular ends is 1/4 of the total distance. What is the distance between the two parallel sides of the track? Express your answer as a decimal to the nearest hundredth.



29.__________ What is the sum of the coordinates of the point at which $y = x - 3$ and $y = -2x + 9$ intersect?

30.__________ A spinner is divided into three congruent sections colored red, blue and green. The numbers 1 through 6 appear on the faces of a fair number cube. When the pointer on the spinner is spun and the number cube is rolled, what is the probability that the pointer lands within the blue section and an even number is rolled? Express your answer as a common fraction.





Warm-Up 3

31. ______ What is the value of $\frac{6!}{5!+4!}$?

32. ______ The same digit A occupies both the thousands and tens places in the five-digit number 1A,2A2. For what value of A will 1A,2A2 be divisible by 9?

33. ______ pounds In Lewis Carroll's *Through the Looking-Glass*, this conversation takes place between Tweedledee and Tweedledum. Tweedledum says, "The sum of your weight and twice mine is 361 pounds." Tweedledee answers, "The sum of your weight and twice mine is 362 pounds." What is the absolute difference in the weights of Tweedledee and Tweedledum?

34. ______ pieces A 63-inch long string is cut into pieces so that their lengths form a sequence. First, a 1-inch piece is cut from the string, and each successive piece that is cut is twice as long as the previous piece cut. Into how many pieces can the original length of string be cut in this way?

35. ______ Two fair, six-sided dice are rolled. They are marked so one die has the numbers 1, 3, 5, 7, 9, 11 and the other has the numbers 2, 4, 6, 8, 10, 12. What is the probability that the sum of the numbers rolled is divisible by 5? Express your answer as a common fraction.

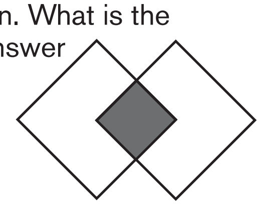
36. $ ______ A shirt company charges a one-time setup fee plus a certain price per shirt. An order of 10 shirts costs $84. An order of 20 shirts costs $159. How much does an order of 30 shirts cost?

37. ______ On a number line, what is the nearest integer to π^2?

38. ______ minutes Cara needs to drive to Greenville. She can drive 50 mi/h along a 200-mile highway, or she can take a different route, which requires her to drive 150 miles at 60 mi/h and then 50 miles at 40 mi/h. How many minutes would Cara save by taking the faster route?

39. $ ______ Square tiles with 4-inch sides are 20¢ each, and square tiles with 6-inch sides are 40¢ each. How much will Jerry save tiling a 4-foot by 6-foot floor with the 6-inch tiles laid side by side instead of the 4-inch tiles laid side by side?

40. ______ units^2 Two unit squares intersect at the midpoints of two adjacent sides, as shown. What is the area of the shaded intersection of the two square regions? Express your answer as a common fraction.



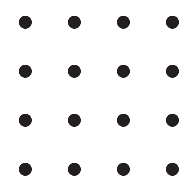
Warm-Up 4

41. ______ If $a - b = 0$, what is the value of $a \times b$? Express your answer in terms of a.

42. ______ When rolling two fair, eight-sided dice, each with faces numbered 1 through 8, what is the probability that the two numbers rolled have a sum of 9? Express your answer as a common fraction.

43. ______ If $2015 = 101a + 19b$, for positive integers a and b, what is the value of $a + b$?

44. ______ squares How many squares can be drawn using only dots in this grid of 16 evenly spaced dots as vertices?



45. ______ ounces A cup contains 6 ounces of milk. Two ounces of chocolate syrup are added to the cup and thoroughly mixed. Then 2 ounces of that mixture are poured out. How many ounces of chocolate syrup are in the remaining mixture? Express your answer as a mixed number.

46. ______ If $f(x) = \sqrt{x+4}$, for what value of x does $f(x) = 3$?

47. ______ What is the product of the greatest common factor and least common multiple of 48 and 72?

48. ______ trips There are 13 stations along the Cheshire Railroad, which runs in a straight line from east to west. A "trip" is defined by its starting and ending stations (regardless of intermediate stops) and must always go westward. How many different trips are possible along the Cheshire Railroad?

49. ______ units² What is the area of $\triangle ABC$ with vertices A(2, 3), B(17, 11) and C(17, 3) ?

50. ______ If x is positive, what is the result when $8x$ is doubled and then divided by one half of $8x$?

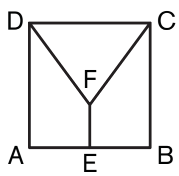
Workout 2

51. ________ points In a basketball game, Aisha and Britney scored 23 points in all; Aisha and Courtney scored 21 points in all; and Britney and Courtney scored 20 points in all. How many points did the three girls score altogether?

52. ________ % The peak of volcano Mauna Kea is 13,803 feet above sea level. When measured from its oceanic base, it measures 33,100 feet vertically to its peak. What percent of Mauna Kea's altitude is below sea level? Express your answer to the nearest whole number.

53. ________ inches A pizza of diameter 16 inches has an area large enough to serve 4 people. What is the diameter of a pizza with an area large enough to serve 12 people? Express your answer as a decimal to the nearest tenth.

54. ________ cm



Square ABCD has sides of length 12 cm. The three interior segments divide the square, as shown, into two congruent trapezoids and an isosceles triangle, all with equal areas. What is the length of segment CF?

55. ________ Given the following facts about the integers *a*, *b*, *c*, *d*, *e* and *f*, what is the value of *a* if $0 \le a \le 60$?

a is odd. $c = \frac{b}{2}$ is even. $e = \frac{d-1}{2}$ is odd.

$b = \frac{a-1}{2}$ is even. $d = \frac{c}{2}$ is odd. $f = \frac{e-1}{2}$ is even.

56. $ ________ The box office staff at a theater sets the ticket price so that total ticket sales will be $7200 if tickets are sold for all 240 seats. By what amount should they increase ticket prices if they want to keep total ticket sales at $7200 but they sell 15 fewer tickets?

57. ________ minutes Lindsey's e-mail account was bombarded with 34,000 spam messages. The web interface allows Lindsey to delete batches of 50 messages at one time, a process that takes 0.5 second to complete. What is the fewest number of minutes will it take Lindsey to delete all 34,000 messages if she deletes batches of 50 messages at a time? Express your answer to the nearest whole number.

58. ________ Let n represent the smallest positive integer such that $2015 + n$ is a perfect square. Let m represent the smallest positive integer such that $2015 - m$ is a perfect square. What is the value of $n + m$?

59. ________ Emilio started a job on July 1, 2000 at a salary of $40,000. If he got an increase of 10% each year, in what year did his salary first exceed $80,000?

60. ________ inches Anne is trying out for the track team, and she had three triple jump attempts. The distance of each triple jump attempt is measured to the nearest inch, and her first two attempts measured 36' 4" and 38' 4", respectively. If Anne's third attempt measured no less than her first attempt and no more than her second attempt, what is the difference between the greatest and least possible averages for the three triple jump attempts?

Warm-Up 5

61. $________ Rachel, Siriana, Tanya and Ursula each contributed to a lottery pool. Rachel contributed $12, Siriana $15, Tanya $20 and Ursula $8. One of their tickets was a winner, paying $1100. If the winnings were distributed in proportion to each person's contribution, what was Siriana's share of the winnings?

62. ________ ways The Statesville Middle School basketball team has 8 players. If a player can play any position, in how many different ways 5 starting players be selected?

63. ________ What is 5 times the sum of all the distinct positive factors of 144?

64. ________ The denominator of a fraction is 2 more than its numerator. The reciprocal of this fraction is equal to the fraction itself. What is the sum of its numerator and denominator?

65. ________ Julius has some spare change consisting of quarters, dimes and nickels. If the ratio of quarters to dimes is 3:4 and the ratio of quarters to nickels is 4:5, what is the ratio of dimes to nickels? Express your answer as a common fraction.

66. ________ miles When Mr. Tesla drives 60 mi/h, the commute from home to his office takes 30 minutes less than it does when he drives 40 mi/h. What is the distance of Mr. Tesla's commute from home to his office?

67. ________ hours Jack Frost makes 18 snowballs every hour, but 2 snowballs melt each 15 minutes. How many hours will it take Jack to accumulate 2 dozen snowballs? Express your answer as a decimal to the nearest tenth.

68. ________ years old A mother was 26 years old when her daughter was born. The mother is now 6 years less than 3 times as old as her daughter. How old is the daughter now?

69. ________ red marbles A bag contains a total of 36 marbles colored either red or blue. Twice the number of red marbles is 6 less than the number of blue marbles. How many red marbles are in the bag?

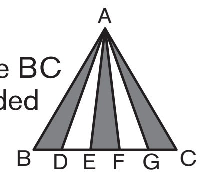
70. ________ When $a > 5$ and $b \leq 5$, $a @ b = (a + b)(a - b)$, and when $a \leq 5$ and $b > 5$, $a @ b = (b + a)(b - a)$. What is the value of $2 \times ((2 @ 6) @ -4) - 1$?

Warm-Up 6

71.________ If $\frac{1}{n} + \frac{1}{2n} + \frac{1}{3n} = k$, what is the value of nk? Express your answer as a common fraction.

72.________ minutes Ellie can paint a room in 2 hours, and Diedre can paint the same room in 4 hours. How many minutes will it take Ellie and Diedre, working together, to paint the room?

73.________ cm^2 Equilateral triangle ABC has sides of length 10 cm. If D, E, F and G divide base BC into five congruent segments, as shown, what is the total area of the three shaded regions? Express your answer in simplest radical form.



74.________ degrees Given that the Earth is turning at a constant rate around its axis and it makes exactly one complete rotation every 24 hours, how many degrees does the center of home plate at Fenway Park rotate around Earth's axis from 6 a.m. on May 8th to noon on May 10th of the same year?

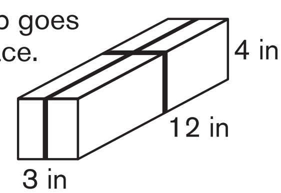
75.________ sides If the difference in the degree measure of an interior and exterior angle of a regular polygon is 100°, how many sides does the polygon have?

76.________ What positive four-digit integer has its thousands and hundreds digits add up to the tens digit, its hundreds and tens digits add up to its ones digit and its tens and ones digits add up to the two-digit number formed by the thousands and hundreds digits?

77.________ meters The surface area of a sphere, in square meters, and its volume, in cubic meters, are numerically equal. What is the length of the radius of the sphere?

78.________ slips Ben won a writing contest at school. To determine his prize, he draws slips of paper from a bag containing 16 slips, with the names of 4 different prizes written on 4 slips each. He continues to draw slips without replacement until he has selected 4 slips for the same prize. What is the maximum number of slips that Ben can draw without knowing his prize?

79.________ inches The wrapped present shown has two loops of ribbon around it. Each loop goes completely around the box once and always runs down the middle of a face. The two loops overlap each other at a right angle, but the ends of the ribbon do not overlap when making a loop. How many inches of ribbon were used?



80.________ times On a 12-hour digital clock, at how many times during a 24-hour day will all of the digits showing the time be the same?

Workout 3

81. ________ If $(x + y)^2 = x^2 + y^2$, what is the value of xy?

82. ________ % Kiera and Aubrey were the only candidates for SGA President. Kiera received 44% of the 7th graders' votes, and Aubrey received 42% of the 8th graders' votes. The student body consists of 325 7th graders and 350 8th graders, and every student voted for one of these two candidates. What percent of the students' votes did the winner receive? Express your answer to the nearest whole number.

83. ________ inches Enoch has a photo that is 8 inches high and 6 inches wide. He wants to enlarge it so that it can fit into a frame that is 10 inches by 7 inches. How much will he need to crop from the width to exactly fill the entire frame? Express your answer as a decimal to the nearest tenth.

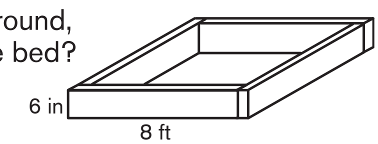
84. ________ miles At 1:00 p.m. an airplane left the local airport and flew due east at 300 mi/h. At 3:00 p.m. a second plane left the same airport and flew due north at 400 mi/h. Assuming the curvature of the earth is negligible, how many miles apart are the planes at 8:00 p.m.?

85. ________ If the mean of the integers 7, 3, 11, 13, 5 and x is 4 more than the mode of the six integers, what is the value of x?

86. ________ students After 6 new students entered the Happy Hearts Childcare Center and 2 students exited, there were 3 times as many students as before. How many students were present before students entered and exited the center?

87. ________ The product of three consecutive prime numbers is 2431. What is their sum?

88. ________ ft^3 Boards that are 8 feet long, 2 inches thick and 6 inches wide are used to make the raised garden bed shown here. If the boards are placed on level ground, how many cubic feet of soil are needed to completely fill the bed? Express your answer to the nearest whole number.



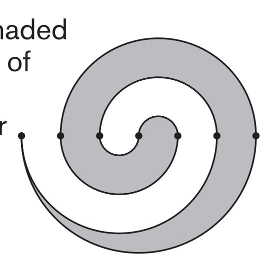
89. ________ dollars Bennie is ordering a new computer. A 6.25% sales tax will be added to the price of the computer, and then an $11 delivery charge will be added to that total. If Bennie has $500 to spend, what is the maximum price of a computer that he can afford? Express your answer to the nearest whole number.

90. ________ teams Each team in the softball league plays each of the other teams exactly once. If 21 games are played, how many teams are in the league?

Warm-Up 7

91. ______ % Mrs. Smith's 1st period class of 28 students averaged 84% on the last test. Her 2nd period class of 24 students averaged 86% on the same test. What must her 3rd period class of 32 students average so that all of the students in her three classes have a combined average of 80%?

92. ______ units² In the figure the collinear dots are equally spaced 2 units apart, and the shaded region is formed from two semicircles of diameter 2 units, two semicircles of diameter 6 units, two semicircles of diameter 10 units and one semicircle of diameter 12 units. What is the area of this shaded region? Express your answer in terms of π.



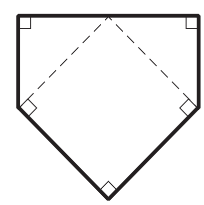
93. ______ If $5a - b - c = 36$ and $b = c = \frac{1}{2}a$, what is the value of a?

94. ______ seats A bus stopped at Main Street and boarded passengers, after which half of its seats were filled. At its next stop, at Oak Street, 2 passengers got off and 7 got on, and then 60% of the seats were filled. How many seats are there on the bus?

95. ______ Three fractions are inserted between $\frac{1}{4}$ and $\frac{1}{2}$ so that the five fractions form an arithmetic sequence. What is the sum of these three new fractions? Express your answer as a common fraction.

96. ______ In the equation 123 × 4A6 = 5B548, what is the value of A × B, the product of the two missing digits?

97. ______ ft² Home plate at a school's baseball field was constructed by adding two right isosceles triangles to a 1-foot by 1-foot square, as shown. What is the area of this home plate? Express your answer as a common fraction.



98. ______ goldfish Elisa had a new fishbowl but no fish, so Kendell gave her half of his goldfish. Elisa's fishbowl was not very large, so she gave half of her new goldfish to Rocky. Rocky kept 8 of the goldfish he was given and gave the remaining 10 goldfish to Aster. What is the difference between Kendell's starting number of goldfish and the number of goldfish Elisa kept for her new fishbowl?

99. ______ teachers In a school that has 20 teachers, 10 teach mathematics, 8 teach social studies and 6 teach science. Two teach both mathematics and social studies, but none teach both social studies and science. How many teach both mathematics and science?

100. ______ A line segment has endpoints $(-5, 10)$ and (a, b). If the midpoint of the segment is $(13, -2)$, what is the absolute difference between a and b?

Warm-Up 8

101. ________ integers For how many positive integers n is it possible to have a triangle with side lengths 5, 12 and n?

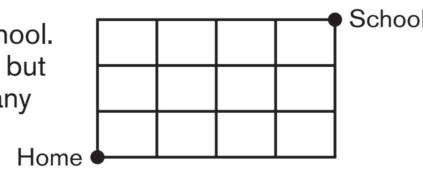
102. ________ laps By the time Shana had completed $\frac{3}{8}$ of her first lap in a race, she had also completed $\frac{1}{32}$ of the entire race. How many laps were there in the race?

103. ________

1	2	3	4	5	6
12	11	10	9	8	7
13	14	15	16	17	18
24	23	22	21	20	19
25	…	…	…	…	…

The counting numbers are written in a table with six columns so that in each successive row the numbers alternate between increasing from left to right and increasing from right to left, as shown. What is the first number in the 15th row?

104. ________ days Alvin lives 4 blocks west and 3 blocks south of his school. He wants to take a different route to school each day, but each route must be exactly 7 blocks long. For how many days can he do this without repeating any route?



105. ________ If x and y are positive integers, and the mean of 4, 20 and x is equal to the mean of y and 16, what is the smallest possible value of $x + y$?

106. ________ cm^2 Five rectangles are arranged in a row. Each rectangle is half as tall as the previous one. Also, each rectangle's width is half its height. The first rectangle is 32 cm tall. What is the sum of the areas of all five rectangles?

107. ________ If one-half of a number is 8 less than two-thirds of the number, what is the number?

108. ________ coins Colton's soda cost \$1.95. He paid for it with nickels, dimes and quarters only. He used 2 more dimes than twice the number of nickels. The number of quarters was 1 more than the number of nickels and dimes combined. How many coins did he use to pay for the soda?

109. \$________ Hiro started a new job with a salary of \$50,000 per year. He received increases of 10%, 20% and 30% at the end of his first, second and third years of employment, respectively. How much did Hiro's salary increase after working three years at his new job?

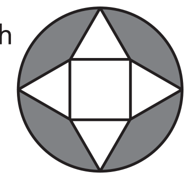
110. ________ A positive integer plus 4 times its reciprocal is equal to the product of the integer and 4 times its reciprocal. What is the integer?

Workout 4

111. ________ cm^2 What is the area of a circle that circumscribes a 6-cm by 8-cm rectangle? Express your answer to the nearest whole number.

112. ________ For positive integer n, $n? = n! \cdot (n - 1)! \cdot \ldots \cdot 1!$ and $n\# = n? \cdot (n - 1)? \cdot \ldots \cdot 1?$. What is the value of $4\# \cdot 3\# \cdot 2\# \cdot 1\#$?

113. ________ % An equilateral triangle is placed on the outside of each side of a square of length 6 cm, and a circle is then drawn through four vertices of the triangles as shown. What percent of the area of the circle is the unshaded region? Express your answer to the nearest whole number.



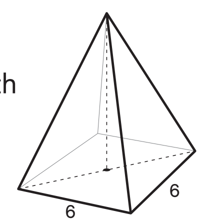
114. ________ If 21 is written as a sum of n consecutive positive integers, what is the greatest possible value of n?

115. ________ minutes Normally, the hose in Elena's garden will fill her small pool in 15 minutes. However, a leak in the hose allows $\frac{1}{3}$ of the water flowing through the hose to spill into the flower bed. How many minutes will it take to fill the pool? Express your answer as a decimal to the nearest tenth.

116. ________ people How many people must be in a group to guarantee that at least 3 of them share the same first initial and last initial?

117. ________ inches The ratio of width to height for Carl's TV screen is 16:9. Carl's TV screen has a 37-inch diagonal. What is the width of his TV screen? Express your answer as a decimal to the nearest tenth.

118. ________ m^3 A pyramid with a square base of side length 6 m has a height equal to the length of the diagonal of the base. What is the volume of the pyramid? Express your answer in simplest radical form.



119. ________ in^3 Boynton's sheet cake measures 18 × 24 inches and has a height of 4 inches. However, these measurements include a $\frac{3}{4}$-inch thick layer of frosting on the top and sides. What is the volume of Boynton's cake excluding the frosting? Express your answer to the nearest whole number.

120. ________ What is the smallest positive integer value of x for which $54x$ is a perfect square?

Warm-Up 9

121. ________ degrees The ratio of the angles of a quadrilateral is 3:4:5:6. What is the number of degrees in the largest of the angles?

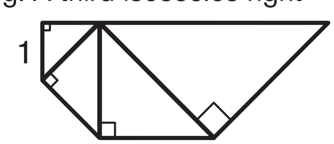
122. ________ Six positive integers have a mean of 6. If the median of these six integers is 8, what is the largest possible value of one of these six integers?

123. $________ The prize money for the Math County Science Fair was divided among the top three projects so that the 1st place winner got as much as the 2nd and 3rd place winners combined, and the 2nd place winner got twice as much as the 3rd place winner. If the total prize money awarded was $2400, how much did the 3rd place winner receive?

124. ________ For the function defined as $f(x) = \begin{cases} x+4 \text{ when } x < -1 \\ x^2 - 6 \text{ when } x \geq -1 \end{cases}$, what is the value of $f(f(2))$?

125. ________ What is the value of $\dfrac{20^2 - 15^2}{18^2 - 17^2}$?

126. ________ units Starting with an isosceles right triangle with legs of length 1 unit, a second isosceles right triangle is built using the hypotenuse of the first triangle as a leg. A third isosceles right triangle is then built using the second triangle's hypotenuse as a leg, and so on, as demonstrated in the figure. If this pattern continues, what will be the number of units in the length of the hypotenuse of the 20th isosceles right triangle?



127. ________ miles Tony's Towing Service charges $30.00 to hook a vehicle to the tow truck and $1.75 for each mile the vehicle is towed. Mr. Alman's car broke down at school and was towed to his house. If the total amount charged by Tony's Towing was $59.75, what is the distance Mr. Alman's car was towed from the school to his house?

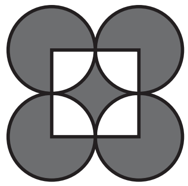
128. $________ A car's present value is $20,000, and its value decreases by the same percentage every year. At the end of one year, it will be worth $18,000. What will it be worth at the end of 3 years?

129. ________ What is the least possible sum of two positive integers whose product is 182?

130. ________ colors Chin-Chin is constructing a tetrahedron and a cube using gumdrops for vertices and toothpicks for edges. She wants to have different-colored gumdrops on the two ends of each toothpick, and no gumdrop used for the cube should be the same color as any gumdrop used for the tetrahedron. What is the least number of colors Chin-Chin needs?

Warm-Up 10

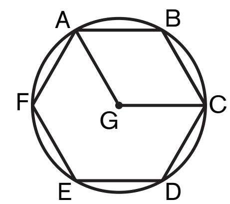
131. ________ mm^2



In the figure shown, four circles of radius 4 mm are centered at the corners of a square of side length 8 mm. What is the total area of the shaded regions? Express your answer in terms of π.

132. ________ Consider any positive three-digit integer that has all of its digits distinct and none equal to zero. What is the largest possible difference between such an integer and any integer that results from rearranging its digits?

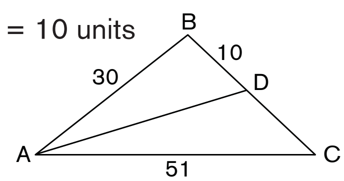
133. ________ $units^2$ A regular hexagon ABCDEF is inscribed in a unit circle with center G, as shown. What is the area of quadrilateral ABCG? Express your answer as a common fraction in simplest radical form.



134. ________ inches The amount of paint required to cover a surface is directly proportional to the area of that surface. The amount of paint needed to cover five spheres of radius 10 inches is the same as the amount of paint required to cover a solid right circular cylinder with radius 20 inches. What is the height of the cylinder?

135. ________ If $x + 2y + 3z = 6$, $2x + 3y + z = 8$ and $3x + y + 2z = 10$, what is the value of $x + y + z$?

136. ________ units In triangle ABC, segment AD bisects angle A. If AB = 30 units, BD = 10 units and AC = 51 units, what is the length of segment BC?



137. ________

1
2
3,4
5,6,7,8,9,10,11

The positive integers are written in order in rows of various lengths. The first row contains the number 1. For every row after the first, the number of entries in the row is the sum of the numbers in the previous row. The first four completed rows are shown. What is the last number in the sixth row?

138. ________ If x and y are negative integers and $x - y = 1$, what is the least possible value for xy?

139. ________ $units^2$ ABCD is a square with side length 4 units, and AEFC is a rectangle with point B on side EF. What is the area of AEFC?

140. ________ liters One hundred liters of a salt and water solution contains 1% salt. After some of the water has evaporated, the solution contains 5% salt. How many liters of water evaporated?

Workout 5

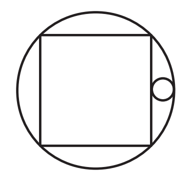
141. ________ The mean of a set of n numbers is 12, and the mean of a set of $3n$ numbers is 6. What is the mean when the two sets are combined? Express your answer as a decimal to the nearest tenth.

142. ________ units² The diagonals of convex quadrilateral ABCD intersect at X. If the areas, in square units, of △AXB, △BXC and △CXD are 8, 15 and 5, respectively, what is the area of △DXA? Express your answer as a decimal to the nearest tenth.

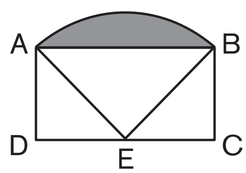
143. ________ What is the sum of all of the four-digit numbers whose digits are permutations of 1, 2, 3 and 4?

144. ________ Duncan picks a number and will subtract 6 each second. Taz picks a different number and will add 8 each second. They begin at the same time, and after 14 seconds, both arrive at the number 25. What is the sum of Duncan's and Taz's starting numbers?

145. ________ units A square is inscribed in a circle of radius 2 units, and then the largest possible circle is inscribed between the square and the original circle, as shown. What is the radius of the inscribed circle? Express your answer as a decimal to the nearest tenth.



146. ________ cm²



In rectangle ABCD, shown here, AB = 4 cm and BC = 2 cm. If E is the midpoint of side DC and also is the center of a circle that contains points A and B, what is the area of the shaded segment of the circle determined by chord AB? Express your answer as a decimal to the nearest tenth.

147. ___:___ a.m. Ellie and Emma live 1.04 miles from each other. They decide to meet by walking toward each other, Ellie at 2.4 mi/h and Emma at 2.8 mi/h. If they both leave at 8:00 a.m., at what time in the morning will they meet?

148. ________ What is the smallest absolute difference between the squares of two distinct positive four-digit integers?

149. ________ bowls At the school fair, MATHCOUNTS parents sold chocolate and vanilla ice cream as a fund-raiser. Forty bowls of chocolate ice cream were sold for $2.15 per bowl. Bowls of vanilla ice cream sold for $1.90 each. How many bowls of ice cream were sold if the total amount of money collected was $158.20?

150. ________ Driving at an average speed of 66 mi/h, Dusty traveled m miles in n minutes. What is the least possible whole-number value of n such that the value of m also is a whole number?

Warm-Up 11

151. ________ times The indicator lights on two different pieces of machinery blink at different intervals, one every 4 seconds and the other every 7 seconds. If they blink together at 10:00 p.m., how many more times will they blink together before 10:15 p.m.?

152. ________ base 10 What is the absolute difference, expressed in base 10, between the largest three-digit base 5 number and the smallest four-digit base 4 number?

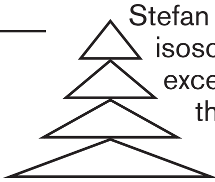
153. ________ The Chug-A-Long Train Company boxes toy trains with either 5 cars or 7 cars per box. The trains in stock have a total of 53 cars. If Charles selects a box at random, what is the probability that the box contains 7 cars? Express your answer as a common fraction.

154. ________ Given the following facts about the numbers a, b, c and d, what is the value of $a + b$? Express your answer as a mixed number.

$ab = 1$
$bc = -9$
$b + c + d = 0$
$b = -c$
$c < -a$

155. ________ units² Twenty-seven unit cubes are arranged to form a $3 \times 3 \times 3$ cube. The center unit cube from each face is then removed. What is the surface area of the resulting solid?

156. ________

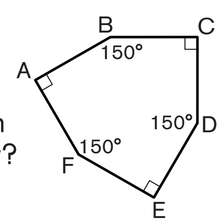


Stefan created this tree design in his computer drawing class. It consists of four isosceles triangles of the same height arranged vertically as shown. With the exception of the top triangle, the apex of each triangle is the midpoint of the base of the triangle above it, and the base of each triangle is 50% larger than the base of the triangle above it. What is the ratio of the area of the smallest triangle to the area of the largest? Express your answer as a common fraction.

157. (____ , ____) What are the coordinates of the point at which the line through the points (2, 6) and (5, 9) intersects the line through the points (−1, −1) and (5, −7)? Express your answer as an ordered pair.

158. ________ If a and b are positive integers such that $ab = 48$ and $a - b = 8$, what is the value of $a + b$?

159. ________ As shown, convex hexagon ABCDEF has right angles at A, C and E, and 150-degree angles at B, D and F. If each side is 2 inches long, its area can be expressed in simplest radical form as $p + q\sqrt{3}$. What is the value of pq?



160. ________ integers How many integers from 100 to 999 have three different digits?

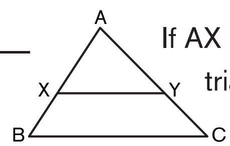
Warm-Up 12

161.________ What is the value of $\dfrac{2 \times 4 \times 6 \times 8 \times 10 \cdots \times 20}{10!}$?

162.________ What is the sum of all positive integers less than 20 that cannot be written as the sum of two prime numbers?

163.________ cm A circle has a diameter of 8 cm. A chord perpendicular to its diameter divides the diameter into segments of lengths 1 cm and 7 cm. What is the length of the chord? Express your answer in simplest radical form.

164.________

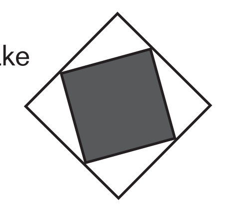


If AX and AY are $\frac{2}{3}$ of AB and AC, respectively, what is the ratio of the area of triangle AXY to trapezoid XYCB? Express your answer as a common fraction.

165.________ integers The *digital sum* of a number is the sum of its digits. How many positive three-digit integers have a digital sum of 5?

166.________ mi/h The Zed Zee courier drove to a location 180 miles away to deliver an urgent package. The courier's average speed driving from the delivery location back to her starting location was 20 mi/h less than her average speed driving to the delivery location. If the entire trip took 7.5 hours, what was her average speed driving to the delivery location?

167.________ One square is inscribed in another so that the sides of the inner square make 30-degree and 60-degree angles with the sides of the outer square. Each side of the inner square is 4 units, and the area of the outer square, in simplest radical form, is $a + b\sqrt{3}$. What is $a + b$?



168.________ A set is said to be *closed* under multiplication if the product of elements in the set also is an element in the set. For what number k is the set $\{0, -1, k\}$ closed under multiplication?

169.________ units³ A certain box of width w has a length that is twice its width, and its height is three times its width. What is the total volume of 24 of these boxes? Express your answer in terms of w.

170.________ If $a_1 = 13$ and $a_n = 77$, for an arithmetic sequence of integers, $a_1, a_2, a_3, \ldots, a_n$, with n terms, what is the median of all possible values of n?

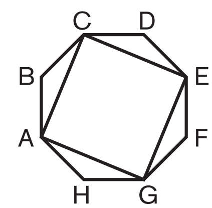
Workout 6

171. ________ 1s Becky tries an experiment. She writes some numbers on the blackboard and then applies the following rule: she picks any number on the board that is greater than 1, erases it and replaces it with the list of its proper divisors. For example, if the number 6 was on the board, she would apply the rule by erasing the 6 and replacing it with the numbers 1, 2 and 3. The experiment ends when there are only 1s left on the board. If Becky begins with just the number 72 on the board, how many 1s will be on the board when she is finished?

172. ________ mm A bug is walking on the ticking second hand of a clock, starting from the center and walking outward. Every second, the bug walks 1 mm along the stationary second hand, and then the hand ticks while the bug stands still. If the bug starts at the very center of the clock and proceeds for exactly 60 seconds, what is the total distance that the bug will travel? Express your answer as a decimal to the nearest tenth.

173. ________ seconds A Ferris wheel has the same height as a building with 60 floors of identical height. After boarding at the bottom of the Ferris wheel, Courtney used a stopwatch to find that it took 8 minutes 26 seconds to rise to the top of the 45th floor of the building. How many seconds will it take from there for the Ferris wheel to bring her back around to where she started, assuming the wheel rotates at a constant rate?

174. ________ units² Regular octagon ABCDEFGH has side-length 1 unit. What is the area of square ACEG? Express your answer as a decimal to the nearest tenth.



175. ________ strings How many distinct three-letter strings can be formed using three of the five letters in the word *SILLY*?

176. ________ bunnies Sammie took his little sister to the petting zoo. His sister really liked the area with chicks and bunnies. Altogether, in the group of 27 chicks and bunnies, Sammie counted 78 legs. Assuming every chick had two legs and every bunny had four legs, how many bunnies were in the group?

177. ________ % A path crosses a rectangular field on a diagonal. If someone travels across the field on the diagonal, instead of walking along the sides, what is the greatest possible percent reduction in total distance traveled? Express your answer to the nearest whole number.

178. ________ cm³ For a particular rectangular solid with integer dimensions, the sum of its length, width and height is 50 cm. What is the absolute difference between the greatest possible volume and the least possible volume of the solid?

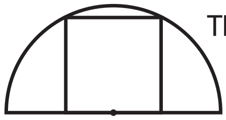
179. ________ A jar contains 28 red jelly beans, 14 black jelly beans and 6 green jelly beans. What is the probability that two jelly beans selected at random, and without replacement, from this jar are the same color? Express your answer as a common fraction.

180. ________ What is the units digit of the product $2^{2015} \times 7^{2015}$?

Warm-Up 13

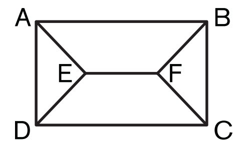
181.______ perfect squares How many perfect squares are factors of 12!?

182.______



The figure shows a square inscribed in a semicircle. What is the ratio of the radius of the semicircle to the side length of the square? Express your answer as a common fraction in simplest radical form.

183.______ units^2 In the figure shown, AE = ED = EF = BF = CF = 1 unit, and $m\angle AED = m\angle BFC = 90$ degrees. What is the area of rectangle ABCD? Express your answer in simplest radical form.



184.______ What is the decimal difference between 1111_3 and 1111_2?

185.______ inches A candle that burns at a uniform rate was 11 inches tall after burning for 4 hours and 8 inches tall after burning for a total of 6 hours. How many inches tall was the candle before it was lit?

186.______ minutes The express train from Addington to Summit travels the 18-mile route at an average speed of 72 mi/h, stopping only in Summit. The local train stops for 1.5 minutes at each of 6 stops between these two locations, and it averages 54 mi/h while it is in motion. How many minutes more does the local train take for this trip than the express train?

187.______ Jack randomly chooses one of the positive integer divisors of 20, and Jill randomly picks one of the positive integer divisors of 30. What is the probability that Jack and Jill pick the same number? Express your answer as a common fraction.

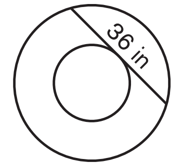
188.______ miles One dragonfly flew in a straight path at a rate of 36 mi/h for 45 minutes. Meanwhile, a second dragonfly rode for 45 minutes on the windshield of a car that was driving in a straight path at 60 mi/h. How many more miles did the second dragonfly travel than the first dragonfly?

189.______ The sum of two numbers is 1, and the absolute difference of the two numbers is 2. What is the product of the two numbers? Express your answer as a common fraction.

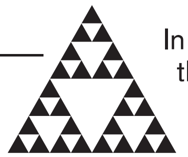
190.______ If $(2a - 3b)^{\sqrt{x}} = 16a^4 - 96a^3b + 216a^2b^2 - 216ab^3 + 81b^4$, what is the value of x?

Warm-Up 14

191. ________ in^2 Two concentric circles and a line segment tangent to the smaller circle are shown. If the length of the line segment is 36 inches, what is the area of the region between the circles? Express your answer in terms of π.



192. ________

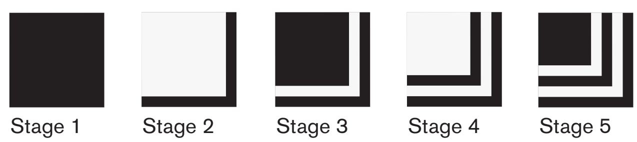


In the equilateral triangle, each downward facing white triangle has its vertices at the midpoints of the sides of the larger upward facing triangle that just contains it. What fraction of the entire figure is white? Express your answer as a common fraction.

193. ________ For non-negative integers m and n, $\frac{m+n}{m-n} = \frac{25}{4}\left(\frac{m-n}{m+n}\right)$ and $m > n$. What is the value of $\frac{m}{n}$? Express your answer as a common fraction.

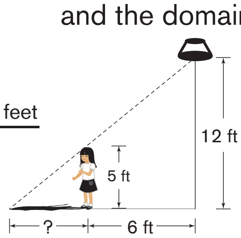
194. ________ cm^3 How many cubic centimeters of silver are needed to cover just the faces of a cube with a layer of silver that is 1 mm thick if the cube has edges of length 6 cm? Express your answer to the nearest whole number.

195. ________ Stage 1 of a pattern is a black square of side length 2015 units. At stage 2, a white square of side length 2014 units is placed on top of the square in stage 1 and positioned so that the upper left vertices of the squares coincide. The pattern continues with alternating white and black squares placed and positioned in this manner at each stage. Each new square placed has sides 1 unit shorter than the square placed at the previous stage. The first five stages are shown. At stage 2015, what portion of the figure will be black? Express your answer as a common fraction.



196. ________ integers The domain of $f(x) = x^2 - 3$ is $\{-4, -3, \ldots, 3, 4\}$. How many integers are in both the range and the domain of f?

197. ________ feet



The lamp of a streetlight is 12 feet above the street below. A girl who is 5 feet tall stands at a point 6 feet from the spot directly below the light. How long is her shadow? Express your answer as a mixed number.

198. ________ integers How many positive integers have the same digits in the same order when written in base 7 and in base 13?

199. ________ Maggie writes the numbers 1, 2, 3, …, 10 on separate slips of paper and tosses the 10 slips into a hat. She then randomly pulls three slips from the hat at the same time. What is the probability that the arithmetic mean of the three selected values is in fact written on one of those same three slips? Express your answer as a common fraction.

200. ________ If $2015 + a = b$ for positive integers a and b, both of which are palindromes, what is the smallest possible value of a?

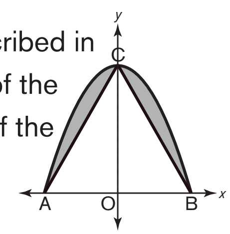
Workout 7

201. ________ If a, b, c and d are positive integers such that $a^b c^d = 2^{10} \times 7^9$, what is the least possible value of $a + b + c + d$?

202. ________ What common fraction is equivalent to $0.2\overline{36}$?

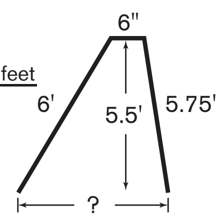
203. ________ units A 3-4-5 right triangle made of paper is cut along the altitude from the right angle, resulting in 2 smaller right triangles. These 2 triangles are then cut along the altitudes from their right angles, then the 4 resulting right triangles are cut the same way and finally, 8 triangles are cut the same way, resulting in 16 smaller right triangles. What is the sum of the perimeters of these 16 triangles? Express your answer as a decimal to the nearest tenth.

204. ________ units^2 The figure shows $\triangle ABC$, with base length b = AB and height h = CO, inscribed in the region bounded by the curve $y = -\frac{1}{4}x^2 + 16$ and the x-axis. If the area of the entire region bounded by the curve and the x-axis is $\frac{2}{3}bh$, what is the area of the shaded region? Express your answer as a decimal to the nearest tenth.



205. ________ What is the value of $1^2 - 2^2 + 3^2 - 4^2 + \ldots + 99^2 - 100^2$?

206. ________ feet



A ladder is 5.5 feet high when opened. The platform at the top is 6 inches wide and parallel to the floor, and the two supports are 6 feet and 5.75 feet, as shown. How many feet long apart are the two supports on the floor? Express your answer as a decimal to the nearest hundredth.

207. ________ The product of three consecutive odd integers is between 64,000,000 and 65,000,000. What is the greatest of the three integers?

208. ________ What is the 2015th digit after the decimal point in the decimal representation of $\frac{1}{13}$?

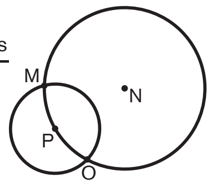
209. ________ sprinklers Jason has a square garden with an area of $\frac{1}{10}$ acre. He is installing sprinklers that each water a circular area with a radius of 11 feet. Jason does not want water from the sprinklers to extend beyond the perimeter of the garden. He also does not want any portion of the garden to receive water from more than one sprinkler, though he realizes that by doing so some portions of the garden will not be watered. Given that an acre is equivalent to 43,560 ft^2, what is the maximum number of sprinklers Jason can install?

210. ________ years A certain computer program will take 2000 years to run using current technology. Every year, advances in technology make it possible to run the program in half the time it would have taken starting in the previous year. However, once the program is started it cannot be interrupted to apply newer technology. Including the years spent waiting to start, what is the least number of years it will take to finish running the program? Express your answer to the nearest whole number.

Warm-Up 15

211. ________ ways In how many ways can the vertices of an equilateral triangle be colored with four available colors? Two colorings are considered the same if they can be obtained from each other by any combination of rotations and reflections.

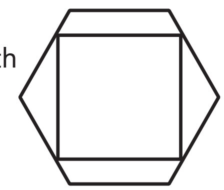
212. ________ degrees



Circle P has its center on circle N. The central angle formed by the radii of circle N that intersect M and O, the points of intersection of the circles, has measure 64 degrees. What is the measure of the central angle formed by the radii of circle P that intersect points M and O?

213. ________ Given that $f(x) = 3x - 7$ and $g(x) = x^2 - 4$, what is the value of $f(g(f(3)))$?

214. ________ cm A square has its four vertices on the sides of a regular hexagon with side length 1 cm. What is the side length of the square? Express your answer in simplest radical form.

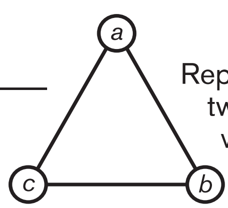


215. ________ % The original price for a pair of shoes was increased by 150%, and then this new price was decreased by 75%. By what percent must the current price be increased to return to the original price?

216. ________ Let P(n) denote the probability that a randomly selected n-digit number contains the digits 42, adjacent and in that order, among its digits. Two 4-digit examples are 3<u>42</u>2 and <u>42</u>05. What is the absolute difference between P(2) and P(3)? Express your answer as a common fraction.

217. ________ % Each year for the first five years of life, a baby elephant's weight increases by 10%. By what percent of its birth weight does an elephant's weight increase during these five years? Express your answer to the nearest whole number.

218. ________



Replace a, b and c with three different positive integers so that the sum of the two numbers along each side is a perfect square. What is the smallest possible value of the sum $a + b + c$?

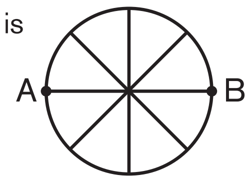
219. $ ________ Ronny had 9 oranges, and Donny had 15 oranges. They met up with Lonny, who had no oranges. Lonny gave $8 to Ronny and Donny, and the three of them shared the oranges equally. If Ronny and Donny split the $8 in proportion to the number of oranges each contributed, how much of the $8 should Ronny receive?

220. ________ The point (8, k) in the first quadrant is the same distance from the point (0, 4) as it is from the x-axis. What is the value of k?

Warm-Up 16

221. ________ If the counting numbers are written in order, what is the value of the 2015th digit written?

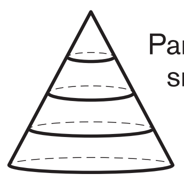
222. ________ cm The circle shown has four equally-spaced diameters of length 2 cm. What is the length of the longest path that can be drawn in one continuous pen stroke from A to B without retracing and without having the path cross itself? (The path may meet itself only at the center but may not cross over itself.) Express your answer in terms of π.



223. ________ cm Two chords of a circle intersect. The point of intersection divides the first chord into two segments of length 5 cm and 8 cm and divides the second chord into two segments, one which has length 4 cm. How long is the second chord?

224. ________ vertices A soccer ball is a polyhedron comprised of 12 pentagons and 20 hexagons. How many vertices does a soccer ball have?

225. ________



Parallel planes divide a cone into a smaller cone and three frustums so that the smaller cone and the three frustums have equal heights, as shown. What is the ratio of the volume of the smallest frustum to the volume of the largest frustum? Express your answer as a common fraction.

226. ________ All points with coordinates (x, y) that are equidistant from the points (1, 3) and (7, 11) lie along a single line. When the equation of the line is written in the form $y = mx + b$, what is the value of b?

227. ________ A sequence begins 1, 2, …, and each term after the second term is the sum of all preceding terms. What is the 15th term of this sequence?

228. ________ Abe chooses a number from Group A, Bob chooses a number from Group B and Charlie chooses a number from Group C.

Group A:	741	345	624	813
Group B:	519	825	717	456
Group C:	134	260	503	152

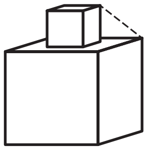
Then Abe chooses a digit X from his number, Bob chooses a digit Y from his number and Charlie chooses a digit Z from his number. The digits are arranged to form the three-digit number XYZ. Abe, Bob and Charlie, in that order, then choose different digits from their selected numbers to form a second three-digit number. Finally, in the same order, they use the remaining digits to form a third three-digit number. What is the sum of the three numbers that are formed?

229. ________ If $6^{12} = 6(6^n + 6^n + 6^n + 6^n + 6^n + 6^n)$, what is the value of n?

230. ________ ways Donald has nine one-day passes to Dizzyworld. He can go alone and use one, or he can take a friend and use two. If he visits every day until he uses all the passes, in how many different ways can he use them? Using two a day for four days and then going alone at the end (2, 2, 2, 2, 1) is different from reversing the order (1, 2, 2, 2, 2).

Workout 8

231. ______ inches



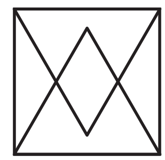
A cube with 2-inch edges is centered on a cube with 4-inch edges so that corresponding edges are parallel, as shown. What is the distance from a top vertex of the smaller cube to the nearest vertex of the larger one? Express your answer in simplest radical form.

232. $ ______

Writing down the four-digit dollar amount of a deposit, a bank teller accidentally transposed two adjacent digits. What is the greatest possible value of the resulting difference between the correct amount and the incorrect amount?

233. ______

Two equilateral triangles are drawn in a square. Two opposite sides of the square coincide with one side of each of the triangles, as shown. What is the ratio of the area of the center rhombus to the area of the entire square? Express your answer as a decimal to the nearest thousandth.



234. ______ paths

B	A	N
A	N	A
N	A	N

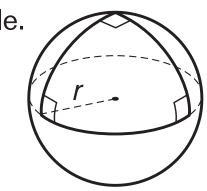
How many paths through this grid spell the word *BANANA*? A path consists of a series of moves from one cell to any cell with which it shares a side and may include multiple moves to a single cell.

235. ______

On average, Josiah allows one hit for every five pitches. If he were to throw nine pitches, what is the probability that he would allow exactly three hits? Express your answer as a decimal to the nearest hundredth.

236. ______

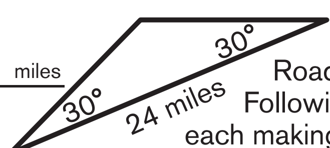
The figure shows a triangular region bounded by three congruent arcs drawn on the surface of a sphere so that the intersection of any two arcs is a right angle. The triangular region's perimeter and surface area have numerical values *A* and *B*, respectively. If $\frac{A^2}{B} = \pi k$, what is the value of *k*? Express your answer as a decimal to the nearest tenth.



237. ______

The two solutions of the equation $x^2 + 7x + 5 = 0$ each can be expressed as a common fraction in simplest radical form as $\frac{l + u\sqrt{m}}{p}$. What is the least value of the product *plum*?

238. ______ miles



Road repairs prevented Anita from taking the 24-mile direct route home. Following a detour, she traveled on two different roads of equal distance, each making a 30-degree angle with the main road, as illustrated. As a result of the detour, how many more miles did Anita travel? Express your answer as a decimal to the nearest tenth.

239. ______ $units^2$

Points A(−1, 2), B(−2, −5) and C(7, −2) are on a circle. What is the area of the circle? Express your answer as a decimal to the nearest tenth.

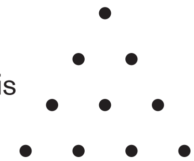
240. ______

The sum of the pairwise products of three consecutive natural numbers is 8111. What is the largest of the three numbers?

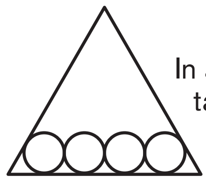
Warm-Up 17

241. ________ In the sum $4 + 2\sqrt{2} + 2 + \sqrt{2} + \ldots$ each term is obtained by dividing the previous term by $\sqrt{2}$. If the sum of the series, in simplest radical form, is $m + n\sqrt{2}$, what is the value of $m + n$?

242. ________ circles How many different circles can be drawn that intersect exactly four points in this triangular grid, made up of 10 points equally spaced?



243. ________ cm



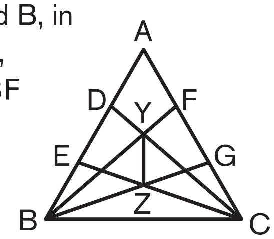
In an equilateral triangle with edge length 12 cm, four congruent circles are tangent to each other and at least one side of the triangle as shown. What is the radius of each circle? Express your answer in simplest radical form.

244. ________ Isosceles triangle ACB has a right angle at C and shares a leg with equilateral triangle BCD of side length 2 in. The triangles, otherwise, do not intersect. Segments BC and AD intersect at E. What is the value of BE/EC? Express your answer in simplest radical form.

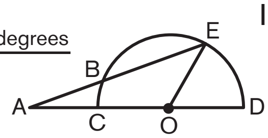
245. ________ What is the value of $a^2(b + c) + b^2(a + c) + c^2(a + b)$ if $a + b + c = 6$, $a^2 + b^2 + c^2 = 40$ and $a^3 + b^3 + c^3 = 200$?

246. ________ days Kevin and Devin each make one hat per day for charity, but they started on different days. Today, Kevin made his 520th hat, and Devin made his 50th. A celebration is planned for the next day that Kevin's hat count is evenly divisible by Devin's hat count. In how many days from today will they celebrate?

247. ________ In equilateral triangle ABC with side length 6 inches, points A, D, E and B, in that order, are equally spaced along side AB, and points A, F, G and C, in that order, are equally spaced along side AC as shown. Segments BF and CD intersect at Y, and segments BG and CE intersect at Z. When expressed as a common fraction in simplest radical form, the length of segment YZ is $\frac{r\sqrt{3}}{s}$ inches. What is the value of $r + s$?



248. ________ degrees



In the figure, $\overset{\frown}{CBD}$ is a semicircle with center O and diameter CD. If AB = OD and the measure of angle EOD is 60 degrees, what is the measure of angle A?

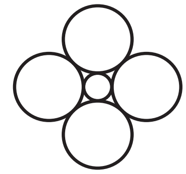
249. ________ times During a game of paintball, ten friends were positioned in a field so that no two of them stood the same distance apart. Each person aimed at his or her closest opponent, and at the signal everyone fired. What is the maximum number of times one player could have been hit?

250. ________ members In a certain state legislature, a proposed bill was defeated with 6 fewer votes for the bill than against it. After the bill was amended, 9 members who had previously voted against the bill were now for it. This resulted in 60% of the legislature now being in favor of the bill. If all those who previously voted for the bill remained in favor of it, how many members are in this state legislature, assuming every member voted each time?

Warm-Up 18

251. ________ units^2 An equiangular hexagon has side lengths 3, 4, 5, 3, 4 and 5 units in that order. What is its area? Express your answer as a common fraction in simplest radical form.

252. ________ mm Each of the four large circles shown here is tangent to two other circles of equal size and is tangent to the center circle, which has radius 1mm. What is the radius of each of the large circles? Express your answer in simplest radical form.



253. ________ units^2 If point Q lies on side AB of square ABCD such that QC = $\sqrt{10}$ units and QD = $\sqrt{13}$ units, what is the area of square ABCD?

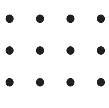
254. ________ If $45_a = 54_b$ for positive integers *a* and *b*, what is the smallest possible value of *a* + *b*?

255. ________ If $\sqrt{2\sqrt[3]{2\sqrt[4]{2}}} = 2^c$, what is the value of *c*? Express your answer as a common fraction.

256. ________ Connecting the centers of the four faces of a regular tetrahedron creates a smaller regular tetrahedron. What is the ratio of the volume of the smaller tetrahedron to the volume of the original one? Express your answer as a common fraction.

257. ________ Daniel began painting a room at 9:00 a.m. Yeong, who can paint twice as fast as Daniel, started helping Daniel at 9:20 a.m., and they worked together until the room was fully painted at 10:00 a.m. What fraction of the room had been painted by 9:30 a.m.? Express your answer as a common fraction.

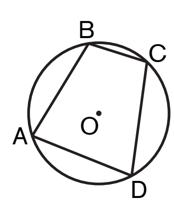
258. ________



Three different dots are randomly chosen from the 16 equally spaced dots in the grid shown. What is the probability that the three dots are collinear? Express your answer as a common fraction.

259. ________ pounds To weigh an object by using a balance scale, Brady places the object on one side of the scale and places enough weights on each side to make the two sides of the scale balanced. Brady's set of weights contains the minimum number necessary to measure the whole-number weight of any object from 1 to 40 pounds, inclusive. What is the greatest weight, in pounds, of a weight in Brady's set?

260. ________ degrees Quadrilateral ABCD is inscribed in circle O as shown. Arc AB = 100 degrees, and arc BC = 50 degrees. What is the measure of angle ADC?

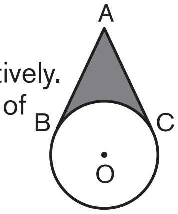


Workout 9

261. ________ If a, b, c and d represent four different positive integers such that $a^2 + d^2 = b^2 + c^2$, what is the least possible value of $|d - a|$?

262. ________ units² The line with equation $ax + by = c$, where a, b and c are positive, forms a right triangle with legs on the x- and y-axes. What is the area of the triangle? Express your answer as a common fraction in terms of a, b and c.

263. ________ in² In the figure, segments AB and AC are tangent to circle O at B and C, respectively. If AB = 12 inches, and minor arc BC measures 120 degrees, what is the area of the shaded region? Express your answer as a decimal to the nearest tenth.



264. ________ Each morning, Khalid and Jack arrive at the dining hall separately, at a random time from 8:00 to 9:00 a.m., and each remains there for exactly 15 minutes while eating. What is the probability they will see each other in the dining hall tomorrow morning? Express your answer as a common fraction.

265. ________ Colby is practicing his basketball shots. He knows that he has a 0.7 chance of making each 2-point shot and a 0.4 chance of making each 3-point shot. If he takes five 2-point and five 3-point shots, what is his probability of earning 20 points or more? Express your answer as a decimal to the nearest thousandth.

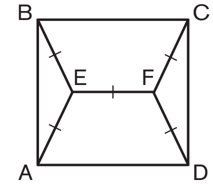
266. ________ in² One circular base of a cylinder with radius 2 inches and height 4 inches is glued flush to the center of each face of a 4-inch cube. What is the surface area of the resulting solid? Express your answer as a decimal to the nearest tenth.

267. ________ fish Sam, Taylor and Pat counted the number of fish in each of their fish tanks. They noticed that Sam's tank had exactly 25% more fish than Taylor's tank, and Pat's tank had exactly 24% more fish than Sam's tank. If each tank had at least one fish, what is the minimum combined number of fish that could have been in the three tanks?

268. ________ degrees

The lateral surface of a right circular cone of radius 48 feet and height 14 feet is cut along a line drawn from the apex perpendicular to the base and unfurled to form the two-dimensional shape shown. What is the central angle measure of the sector that is missing from the circle? Express your answer as a decimal to the nearest tenth.

269. ________ cm² Inside square ABCD, shown here, points E and F are chosen in such a way that AE = BE = CF = DF = EF = 4 cm. What is the area of triangle ABE?



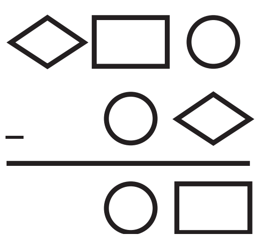
270. ________ Each digit 0 through 9 is used exactly once to create two five-digit numbers. What is the sum of the digits of the greatest product of two such numbers?

Logic Stretch

271.________ Celia, Desi and Everett are each wearing a hat that displays a different whole number from 1 to 9, inclusive. Each number cannot be seen by the person wearing it, but that number is visible to the other two individuals. Everett says, “The sum of the numbers I see is 6.” Celia says, “The product of the numbers I see is 10.” What is the sum of the numbers that Everett could possibly have on his hat?

272.________ people In a survey, 30 people reported that they enjoy some combination of walking, hiking and jogging. The number who enjoy only walking, the number who enjoy only hiking and the number who enjoy only jogging are all equal. Likewise, the number who enjoy only walking and hiking, the number who enjoy only walking and jogging and the number who enjoy only hiking and jogging are equal. In addition, the survey showed that half as many people enjoy exactly two of these activities as those who enjoy only one activity. If three people enjoy all three activities, how many people enjoy jogging?

273.________



In the subtraction problem shown, the shapes ◇, □ and ○ each represent a different digit. What is the value of □◇ ÷ ○?

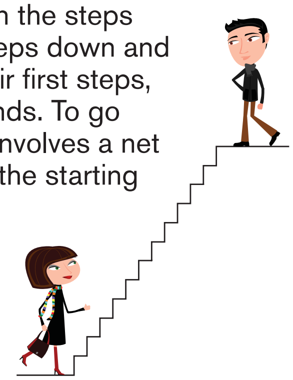
274. Box________ Three identical boxes contain tennis balls, baseballs or both. A label is affixed to each box. The three labels correctly describe the three boxes, but none of the labels is on the correct box. Box 1 is labeled “Tennis Balls.” Box 2 is labeled “Baseballs.” Box 3 is labeled “Tennis Balls & Baseballs.” Devon reaches into Box 3 and pulls out a baseball. Which box contains only tennis balls?

275. Page________ Drew purchased a used 50-page book at the book fair. Drew later realized that the book, in which left-hand pages contained even page numbers and right-hand pages contained odd page numbers, did not contain all 50 pages. The sum of the page numbers on the pages that Drew’s book did contain was 1242. What is the greatest page number that could be on a page missing from Drew’s book?

276.__________ In the addition problem shown, each letter stands for a different digit. If T = 3, what is the value of the four-digit number MATH?

	G	E	T
+	T	H	E
M	A	T	H

277.______ seconds Starting at the lower landing of a staircase, Porscha goes up the steps by repeating a three-step sequence: moving two steps up and then moving one step down. Starting at the upper landing of the same staircase, Micah goes down the steps by repeating a different three-step sequence: moving two steps down and then moving one step up. After simultaneously moving to their first steps, Porscha and Micah both move to another step every 3 seconds. To go from the upper landing to the lower landing of the staircase involves a net movement of 12 steps. How many seconds after moving to the starting steps will Porscha and Micah reach the same step?



278.__________ If the six-digit number 3D6,D92 is divisible by 11, what is the value of D?

279.__________ A special deck of cards contains cards numbered 1 through 4 for each of four suits. Each of the 16 cards has a club, diamond, heart or spade on one side and the number 1, 2, 3 or 4 on the other side. After a dealer mixed up the cards, three were selected at random. What is the probability that of these three randomly selected cards, displayed here, one of the cards showing the number 2 has a heart printed on the other side? Express your answer as a common fraction.

280.__________ The units digit of a three-digit number, ABC, is moved to the left of the remaining two digits to make a new three-digit number, CAB. If CAB − ABC = 162, what is the sum of the least and greatest possible values of ABC?

Solving Inequalities Stretch

QUICK REVIEW OF INEQUALITY PROPERTIES

For any numbers a, b and c,

- if $a > b$, then $a + c > b + c$ and $a - c > b - c$. (applies to $>$, $\geq$, $<$ and $\leq$)
- if $a > b$ and $c > 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$. (applies to $>$, $\geq$, $<$ and $\leq$)
- if $a > b$ and $c < 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$. (applies to $>$, $\geq$, $<$ and $\leq$)
- if $|a| < b$, then $a < b$ and $a > -b$. (applies to $<$ and $\leq$)
- if $|a| > b$, then $a > b$ or $a < -b$. (applies to $>$ and $\geq$)

Solve each inequality, and graph the solution on the number line provided.

281. ______ $3 - \frac{x}{3} \leq 5$

282. ______ $3 - \frac{x}{3} \geq -5$

283. ______ $\left|3 - \frac{x}{3}\right| \leq 5$

284. ______ $3 - \frac{x}{3} < x - 5$

285. ______ $3 - \frac{x}{3} > 5 - x$

286. ______ $\left|3 - \frac{x}{3}\right| < x - 5$

287. ______ $x^2 \leq 25$

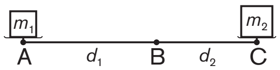
288. ______ $x^2 \geq 25$

289. ______ $x^2 + 4x - 4 > -8$

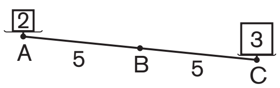
290. ______ $x^2 + 4x - 4 > -7$

Mass Point Geometry Stretch

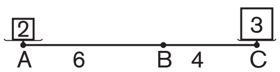
Center of Mass



Consider a seesaw, with a fulcrum at B, that has objects at A and C. As shown, the object at A has a mass of m_1, and its distance from B is d_1. The object at C has mass m_2, and its distance from B is d_2. These examples show how the position of the fulcrum determines whether the seesaw is balanced. The mass at B is $m_1 + m_2$, the sum of the masses at A and C.

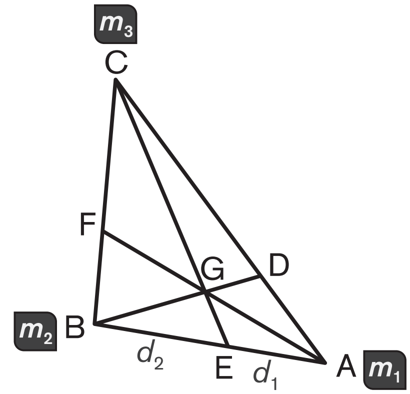


In this first example, B is positioned so that $d_1 = d_2 = 5$. Notice that $m_1 \times d_1 = 2 \times 5 = 10$ and $m_2 \times d_2 = 3 \times 5 = 15$. Although the objects at A and C are equidistant from B, the object at C is lower than the object at A because $m_1 \times d_1 < m_2 \times d_2$.



In this example, B is positioned so that $d_1 = 6$ and $d_2 = 4$. Here, $m_1 \times d_1 = 2 \times 6 = 12$ and $m_2 \times d_2 = 3 \times 4 = 12$. This time, the seesaw is balanced because $m_1 \times d_1 = m_2 \times d_2$. In this case, the position of B is known as the *center of mass*.

A *cevian* is a line segment that joins a vertex of a triangle with a point on the opposite side. Mass point geometry is a technique used to solve problems involving triangles and intersecting cevians by applying center of mass principles. Because triangle ABC, shown here, has cevians AF, BD and CE that intersect at point G, we can apply the center of mass principles presented. For example, side AB is balanced on point E when $m_1 \times d_1 = m_2 \times d_2$.



A *mass point*, denoted *m*P, consists of point P and its associated mass, *m*. Assume point G is the center of mass on which the entire triangle balances. Then the mass at G is the sum of the masses at the endpoints for each cevian and *m*G = *m*A + *m*F = *m*B + *m*D = *m*C + *m*E.

Suppose BF:CF = 3:4 and AD:CD = 2:5, and we are asked to determine the ratios AE:BE, AG:FG and BG:DG.

Start by finding *m*B and *m*C for side BC, which is balanced on point F. We know $m_2 \times 3 = m_3 \times 4$. We can let $m_2 = 4$ and $m_3 = 3$, so 4B + 3C = (4 + 3)F = 7F.

Next, find *m*A for side AC, which is balanced on point D. We know $m_1 \times 2 = m_3 \times 5$. Since $m_3 = 3$, it follows that $m_1 \times 2 = 3 \times 5$ and $m_1 = 15/2$. Rather than having mass point (15/2)A, we can multiply 4B, (15/2)A, 3C and 7F by 2 to get the following mass points: 8B, 15A, 6C and 14F. Now the mass at each point is of integer value.

Now, there is enough information to find *m*D and *m*E, since 15A + 6C = (15+ 6)D = 21D and 15A + 8B = (8 + 15)E = 23E. Therefore, given mass points 15A and 8B, it follows that side AB is balanced on point E when AE:BE = 8:15. In addition, given mass points 21D and 14F, we see that cevians AF and BD both are balanced on point G when AG:FG = 14:15 and BG:DG =21:8.

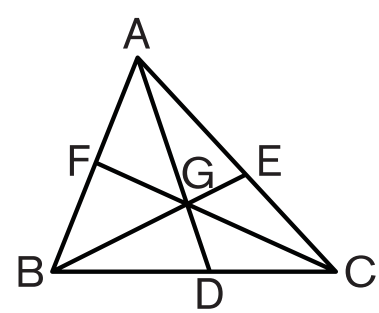
Solve the following problems by using mass point geometry. Express ratio answers as common fractions.

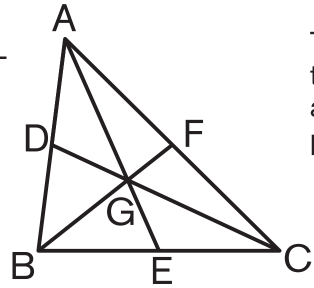
Triangle ABC, shown here, has cevians AD, BE and CF intersecting at point G, with AF:BF = 3:2 and BD:CD = 5:3.

291.__________ What is the ratio of AE to CE?

292.__________ What is the ratio of BG to EG?

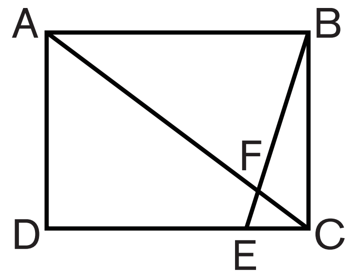
293.__________ What is the ratio of DG to AG?

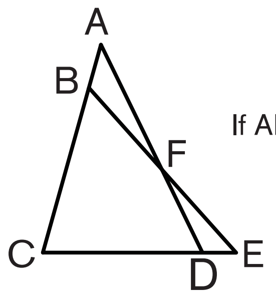


294.__________ 

The medians of a triangle intersect at a point in the interior of the triangle as shown. What is the ratio of the lengths of the shorter and longer segments into which each median is divided at the point of intersection?

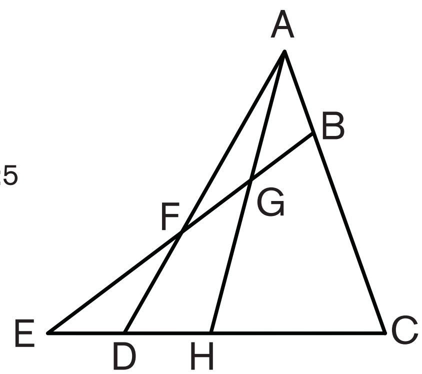
295.__________ In rectangle ABCD, point E is on side DC such that BC = 8, BE = 10 and AC = 17. If segments AC and BE intersect at F, what is the ratio of the area of triangle CFE to the area of triangle AFB?

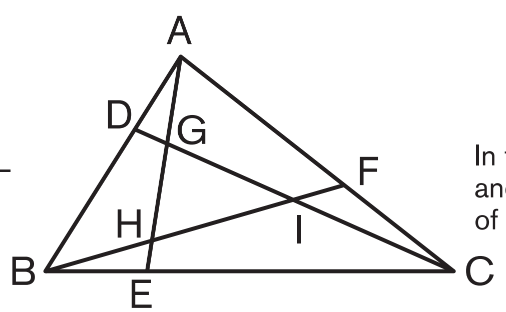


296.__________ 

If AB:BC = 1:4 and AF:DF = 5:4, what is the ratio of DE to CD?

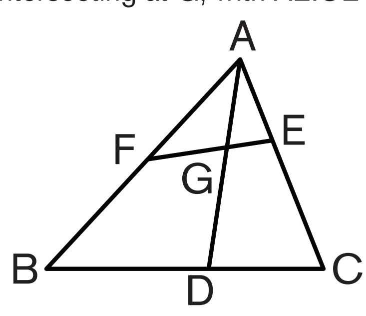
297.__________ For integers *x*, *y* and *z*, if AB:BC = 1:4, AG:GH = 3:5 and AF:DF = 5:4, then CH:DH:DE = *x*:*y*:*z*. What is the value of *x* + *y* + *z*?



298.__________ 

In triangle ABC, AD:BD = 1:2, BE:EC = 1:3 and AF:CF = 3:2. What is the ratio of the area of triangle GHI to the area of triangle ABC?

Triangle ABC, shown here, has cevian AD and transversal EF intersecting at G, with AE:CE = 1:2, AF:BF = 5:4 and BD:CD = 3:2.



299.__________ What is the ratio of AG to DG?

300.__________ What is the ratio of EG to FG?

MATHCOUNTS® COMPETITION SERIES EST. 1983

PROGRAM DETAILS

The MATHCOUNTS Foundation administers its math enrichment, coaching and competition program with a grassroots network of more than 17,000 volunteers who organize MATHCOUNTS competitions nationwide. Each year more than 500 local competitions and 56 “state” competitions are conducted, primarily by chapter and state societies of the National Society of Professional Engineers. All 50 states, the District of Columbia, Puerto Rico, Guam, Virgin Islands and schools worldwide that are affiliated with the U.S. Departments of Defense and State participate in MATHCOUNTS.

This section of the handbook should serve as your main guide for making the most of your time when coaching for competitions. From program rules and procedures to tips from veteran coaches about growing your school’s program, this resource was designed with teachers and coaches in mind.

GETTING STARTED AS A COACH

Thank you so much for serving as a coach in the MATHCOUNTS Competition Series! Your work truly does make a difference in the lives of the students you mentor.

All MATHCOUNTS materials, including this handbook, can be incorporated into regular classroom instruction so that all students learn problem-solving techniques and develop critical thinking skills. If your school’s MATHCOUNTS team meetings are limited to extracurricular sessions, all interested students should be invited to participate—regardless of their academic standing. Because the greatest amount of time in this program will be invested at the school level, having lots of Mathletes participate helps ensure that more students benefit from an experience in MATHCOUNTS.

Generating Student Interest: Here are some tips given to us from successful competition coaches and club leaders for getting students involved in the program at the beginning of the year.

- Ask Mathletes who have participated in the past to talk to other students about participating.
- Ask teachers, parent volunteers and counselors to help you recruit.
- Reach parents through school newsletters, PTA meetings or Back-to-School-Night presentations.
- Advertise around your school by.
  1) posting intriguing math questions (specific to your school) and referring students to the first meeting for answers.
  2) designing a bulletin board or display case with your MATHCOUNTS poster, photos and awards from past years.
  3) attending meetings of other extracurricular clubs (such as honor society) so you can invite their members to participate.
  4) adding information about the MATHCOUNTS team to your school’s website.
  5) making a presentation at the first pep rally or student assembly.

Maintaining a Strong School Program: Having a great start is important, but here are some tips from coaches and club leaders about keeping your program going strong for the entire year.

- Publicize meetings on your school’s website, in a school newsletter and in morning announcements.
- Celebrate your team’s success by
  1) scheduling a special pep rally or awards ceremony for the Mathletes at your school.

2) planning a special field trip, such as to a local college campus or museum, as a reward.
3) planning an end-of-year event, such as a "students vs. teachers" Countdown Round, that gets the entire school involved and recognizes the Mathletes.
4) giving prizes, such as T-shirts, to the students who participate. (You can ask parents and local businesses to help with donations for these.)
5) posting event information, awards won and photos of your team on your school's website.

USING THIS HANDBOOK

Beginning your team meetings in the fall will help you maximize your coaching time and expose your students to more handbook problems before the Chapter Competition.

The *MATHCOUNTS School Handbook* is released every fall and provides the foundation for coaches to prepare their Mathletes to compete in the MATHCOUNTS Competition Series. This resource contains 300 challenging and creative problems that are written to meet the National Council of Teachers of Mathematics Standards for Grades 6-8. The handbook is made available electronically to all U.S. schools through the MATHCOUNTS website, and a hard copy is available upon request to all schools—free of charge. Coaches who register for the MATHCOUNTS Competition Series will receive the handbook in their School Competition Kit.

Handbook Contents and Structure: This handbook consists primarily of three types of math problems—Warm-Ups, Workouts and Stretches—that are designed to complement the Competition Series. All of the problems provide students with practice in a variety of problem-solving situations and may be used to diagnose skill levels, to practice and apply skills or to evaluate growth in skills. The handbook also contains additional resources for coaches to use, as described in the list.

- *Warm-Ups (pp. 9-34)* serve as excellent practice for the Sprint Round and assume that students will not be using calculators. These problems increase in difficulty as students go through the handbook.
- *Workouts (pp. 11-35)* serve as practice for the Target and Team Rounds and assume the use of a calculator. These problems also increase in difficulty as students go through the handbook.
- *Stretches (pp. 36-40)* focus on specific standards and topics and cover a variety of difficulty levels. These problems can be incorporated into your students' practice at any time.
- *Vocabulary and Formulas (pp. 52-53)*
- *Solutions (pp. 54-76)* provide complete explanations for how to solve all problems in this handbook. These are only possible solutions; you or your students may come up with more elegant solutions.
- *Difficulty Rating (p. 77)* explains ratings on a scale of 1-7, with 7 being the most difficult.
- *Answer Key (pp. 77-81)* provides answers for all handbook problems.
- *Common Core State Standards (p. 82)* explain how problems are aligned to the Common Core, as shown in the Problem Index.
- *Problem Index (pp. 83-84)* helps you incorporate handbook problems into your curriculum. This index organizes all 300 problems by topic, difficulty rating and mapping to the Common Core.

Tips for Productive Practices: Here are some suggestions for getting the most out of team meetings.

- Encourage discussion of the problems so that students learn from one another.
- Encourage a variety of methods for solving problems.
- Have students write problems for each other.
- Use the Problem of the Week, posted on www.mathcounts.org/potw every Monday.
- Practice working in groups to develop teamwork (and to prepare for the Team Round).
- Practice oral presentations to reinforce understanding.
- Use the Interactive MATHCOUNTS Platform, which includes current and past materials, as well as features that make it easy to collaborate.
- Provide refreshments and vary the location of your meetings to create an, enjoyable atmosphere.
- Recruit volunteers (such as MATHCOUNTS alumni, high school students, parents, communtiy professionals and reitrees) to serve as assistant coaches.

Suggested Practice Schedule: On average, coaches meet with their students for an hour once a week at the beginning of the year, and more often as the competitions approach. Practice sessions may be held before school, during lunch, after school, on weekends or at other times, coordinating with your school's schedule and avoiding conflicts with other activities.

Designing a schedule for your practices will help ensure you are able to cover more problems and prepare your students for competitions. Handbook Stretches can be used at any time during the fall, but below is the recommended schedule for using Warm-Ups and Workouts if you are participating in the Competition Series.

September 2014	Warm-Ups 1-2	Workout 1
October	Warm-Ups 3-6	Workouts 2-3
November	Warm-Ups 7-10	Workouts 4-5
December	Warm-Ups 11-14	Workouts 6-7
January 2015	Warm-Ups 15-16	Workout 8
	MATHCOUNTS School Competition	
February	Warm-Ups 17-18	Workout 9
	Selection of competitors for Chapter Competition	
	MATHCOUNTS Chapter Competition	

To encourage participation by the greatest number of students, postpone selection of your school's official competitors until just before the Chapter Competition.

ADDITIONAL RESOURCES FOR COACHES

Take advantage of other free resources available to coaches. The more MATHCOUNTS problems your students can work on in the fall, the better they will do in competitions.

Free Web Resources: These resources are available to coaches at no cost.

- *Past MATHCOUNTS Competitions:* the 2014 School, Chapter and State MATHCOUNTS Competitions and Answer Keys. Visit www.mathcounts.org/pastcompetition.
- *MATHCOUNTS Problem of the Week:* weekly set of 3-4 theme-based problems focusing on critical thinking and problem-solving skills. You can access the Problem of the Week Archive with problems starting from 2012. Visit www.mathcounts.org/potw.
- *MATHCOUNTS Minis:* monthly instructional math videos featuring Richard Rusczyk from Art of Problem Solving, explaining how to solve many types of problems. You can access the Minis Archive and choose from over 35 past videos. Visit www.mathcounts.org/minis.
- *Interactive MATHCOUNTS Platform:* forum that contains current and past resources, and lets users discuss problems and get instant feedback on their progress. Visit mathcounts.nextthought.com.

Additional Resources: These resources are available for purchase.

- *MATHCOUNTS OPLET:* The Online Problem Library and Extraction Tool (OPLET) is a database of over 13,000 problems organized by topic and difficulty level. Visit www.mathcounts.org/oplet.
- *MATHCOUNTS Online Store:* Purchase practice books, supplies, prizes and awards for your team. Visit www.mathcounts.org/store. Below is a list of the most popular practice books available.

  2014 Competitions (previous years available while supplies last)
  The Most Challenging MATHCOUNTS Problems Solved (step-by-step solutions to National Competition Sprint and Target Round problems from 2001 to 2010)
  Practice Problems for MATHCOUNTS, Vol. 1
  Practice Problems for MATHCOUNTS, Vol. 2
  The All-Time Greatest MATHCOUNTS Problems

OFFICIAL RULES AND PROCEDURES

The following rules and procedures govern all MATHCOUNTS competitions. The MATHCOUNTS Foundation reserves the right to alter these rules and procedures at any time. **Coaches are responsible for being familiar with the rules and procedures outlined in this handbook.** Coaches should bring any difficulty in procedures or in student conduct to the immediate attention of the appropriate chapter, state or national official. Students violating any rules may be subject to immediate disqualification.

REGISTRATION

The fastest and easiest way to register for the MATHCOUNTS Competition Series is online at www.mathcounts.org/compreg.

For your school to participate in the MATHCOUNTS Competition Series, a school representative is required to complete a registration form and pay the registration fees. A school representative can be a teacher, an administrator or a parent volunteer who has received express permission from his or her child's school administration to register. By completing the Competition Series Registration Form, the coach attests to the school administration's permission to register students for MATHCOUNTS.

School representatives can register online at www.mathcounts.org/compreg or download the Competition Series Registration Form and mail, fax or e-mail a scanned copy of it to the MATHCOUNTS Registration Office. Refer to the Critical 2014-2015 Dates on page 4 of this handbook for registration contact information.

What Registration Covers: Registration in the Competition Series entitles a school to

1) send 1-10 students (depending on number registered) to the Chapter Competition. *Students can advance beyond the chapter level, but this is determined by their performance at the competition.*
2) receive the School Competition Kit, which includes the *2014-2015 MATHCOUNTS School Handbook*, a recognition ribbon for each registered student, 10 student participation certificates and a catalog of additional coaching materials. *Mailings of School Competition Kits will occur on a rolling basis through December 31, 2014.*
3) receive online access to the 2015 School Competition, along with electronic versions of other competition materials, at www.mathcounts.org/competitioncoaches. *Coaches will receive an e-mail notification no later than November 3, 2014 when the 2015 School Competition is available online.*

Your state or chapter coordinator will be notified of your registration, and you then will be informed of the date and location of your Chapter Competition. **If you have not been contacted by mid-January with competition details, it is your responsibility to contact your local coordinator** to confirm that your registration has been properly routed and that your school's participation is expected. Coordinator contact information is available at www.mathcounts.org/competition.

Deadlines: The sooner your Registration Form is received, the sooner you will receive your preparation materials. To guarantee your school's registration, submit your registration by one of the following deadlines:

Early Bird Discount Deadline: November 14, 2014	Online registrations: submitted by 11:59 p.m. PST E-mailed or faxed forms: received by 11:59 p.m. PST Mailed forms: postmarked by November 14, 2014
Regular Registration Deadline: December 12, 2014*	Online registrations: submitted by 11:59 p.m. PST E-mailed or faxed forms: received by 11:59 p.m. PST Mailed forms: postmarked by December 12, 2014

*Late registrations may be accepted at the discretion of the MATHCOUNTS national office and your local coordinators, but acceptance is not guaranteed. If a school's late registration is accepted, an additional $20 processing fee will be assessed.

Registration Fees: The cost of your school's registration depends on when your registration is postmarked/e-mailed/faxed/submitted online. The cost of your school's registration covers the students for the entire Competition Series; there are no additional registration fees for competing at the state or national level. Title I schools (as affirmed by a school's administration) receive a 50% discount off the total cost of their registration.

Number of Registered Students	Registration Postmarked by 11/14/2014	Postmarked between 11/14/2014 and 12/12/2014	Postmarked after 12/12/2014 (with Late Fee)
1 individual	$25	$30	$50
2 ind.	$50	$60	$80
3 ind.	$75	$90	$110
1 team of 4	$90	$100	$120
1 tm. + 1 ind.	$115	$130	$150
1 tm. + 2 ind.	$140	$160	$180
1 tm. + 3 ind.	$165	$190	$210
1 tm. + 4 ind.	$190	$220	$240
1 tm. + 5 ind.	$215	$250	$270
1 tm. + 6 ind.	$240	$280	$300

ELIGIBILITY REQUIREMENTS

Eligibility requirements for the MATHCOUNTS Competition Series are different from requirements for other MATHCOUNTS programs. Eligibility for The National Math Club or the Math Video Challenge does not guarantee eligibility for the Competition Series.

Who IS Eligible:

- U.S. students enrolled in the 6th, 7th or 8th grade can participate in MATHCOUNTS competitions.
- Schools that are the students' official school of record can register.
- Any type of school, of any size, can register—public, private, religious, charter, virtual or homeschools—but virtual and homeschools must fill out additional forms to participate (see p. 46).
- Schools in the 50 U.S. states, District of Columbia, Guam, Puerto Rico and Virgin Islands can register.
- Overseas schools that are affiliated with the U.S. Department of Defense or State can register.

Who IS NOT Eligible:

- Students who are not full-time 6th, 7th or 8th graders cannot participate, even if they are taking middle school math classes.
- Academic centers, tutoring centers or enrichment programs that do not function as students' official school of record cannot register. *If it is unclear whether your educational institution is considered a school, please contact your local Department of Education for specific criteria governing your state.*
- Schools located outside of the U.S. states and not in the territories listed above cannot register.
- Overseas schools not affiliated with the U.S. Department of Defense or State cannot register.

Number of Students Allowed: A school can register a maximum of one team of four students and six individuals; these 1-10 students will represent the school at the Chapter Competition. Any number of students can participate at the school level. Prior to the Chapter Competition, coaches must notify their chapter coordinator to identify which students will be team members and which students will compete as individuals.

Number of Years Allowed: Participation in MATHCOUNTS competitions is limited to three years for each student, but there is no limit to the number of years a student may participate in school-based coaching.

What Team Registration Means: Members of a school team will participate in the Target, Sprint and Team Rounds. Members of a school team also will be eligible to qualify for the Countdown Round (where conducted). Team members will be eligible for team awards, individual awards and progression to the state and national levels based on their individual and/or team performance. It is recommended that your strongest four Mathletes form your school team. Teams of fewer than four will be allowed to compete; however, the Team Score will be computed by dividing the sum of the team members' scores by 4 (see p. 49), meaning teams of fewer than four students will be at a disadvantage. *Only one team (of up to four students) per school is eligible to compete.*

What Individual Registration Means: Students registered as individuals will participate in the Target and Sprint Rounds but not the Team Round. Individuals will be eligible to qualify for the Countdown Round (where conducted). Individuals also will be eligible for individual awards and progression to the state and national levels. A student registered as an individual may not help his or her school's team advance to the next level of competition. *Up to six students may be registered in addition to or in lieu of a school team.*

How Students Enrolled Part-Time at Two Schools Participate: *A student may compete only for his or her official school of record.* A student's school of record is the student's base or main school. A student taking limited course work at a second school or educational center may not register or compete for that second school or center, even if the student is not competing for his or her school of record. MATHCOUNTS registration is not determined by where a student takes his or her math course. If there is any doubt about a student's school of record, the chapter or state coordinator must be contacted for a decision before registering.

How Small Schools Participate: MATHCOUNTS does not distinguish between the sizes of schools for Competition Series registration and competition purposes. Every "brick-and-mortar" school will have the same registration allowance of up to one team of four students and/or up to six individuals. A school's participants may not combine with any other school's participants to form a team when registering or competing.

How Homeschools Participate: Homeschools and/or homeschool groups in compliance with the homeschool laws of the state in which they are located are eligible to participate in MATHCOUNTS competitions in accordance with all other rules. Homeschool coaches must complete a Homeschool Participation Form, verifying that students from the homeschool or homeschool group are in the 6th, 7th or 8th grade and that each homeschool complies with applicable state laws. Forms can be downloaded at www.mathcounts.org/competition and must be submitted to the MATHCOUNTS national office in order for registrations to be processed.

How Virtual Schools Participate: Virtual schools that want to register must contact the MATHCOUNTS national office by December 1, 2014 for specific registration details. Any student registering as a virtual-school student must compete in the MATHCOUNTS Chapter Competition assigned according to the student's home address. Additionally, virtual-school coaches must complete a Homeschool Participation Form verifying that the students from the virtual school are in the 6th, 7th or 8th grade and that the virtual school complies with applicable state laws. Forms can be downloaded at www.mathcounts.org/competition and must be submitted to the national office in order for registrations to be processed.

What Is Done for Substitutions of Students: Coaches determine which students will represent the school at the Chapter Competition. Coaches cannot substitute team members for the State Competition unless a student voluntarily releases his or her position on the school team. Additional requirements and documentation for substitutions (such as requiring parental release or requiring that the substitution request be submitted in writing) are at the discretion of the state coordinator. A student being added to a team need not be a student who was registered for the Chapter Competition as an individual. Coaches cannot make substitutions for students progressing to the State Competition as individuals. At all levels of competition, student substitutions are not permitted after on-site competition registration has been completed.

What Is Done for Religious Observances: A student who is unable to attend a competition due to religious observances may take the written portion of the competition up to one week in advance of the scheduled competition. In addition, all competitors from that student's school must take the Sprint and Target Rounds at the same earlier time. If the student who is unable to attend the competition due to a religious observance (1) is a member of the school team, then the team must take the Team Round at the same earlier time; (2) is not part of the school team, then the team has the option of taking the Team Round during this advance testing or on the regularly scheduled day of the competition with the other school teams. The coordinator must be made aware of the team's decision before the advance testing takes place. *Advance testing will be done at the discretion of the chapter and state coordinators. If advance testing is deemed possible, it will be conducted under proctored conditions.* Students who qualify for an official Countdown Round but are unable to attend will automatically forfeit one place standing.

What Is Done for Students with Special Needs: Reasonable accommodations may be made to allow students with special needs to participate. However, many accommodations that are employed in a classroom or teaching environment cannot be implemented in the competition setting. Accommodations that are not permissible include, but are not limited to, granting a student extra time during any of the competition rounds or allowing a student to use a calculator for the Sprint or Countdown Rounds. *A request for accommodation of special needs must be directed to chapter or state coordinators in writing at least three weeks in advance of the Chapter or State Competition.* This written request should thoroughly explain a student's special needs, as well as what the desired accommodation would entail. In conjunction with the MATHCOUNTS Foundation, coordinators will review the needs of the student and determine if any accommodations will be made. In making final determinations, the feasibility of accommodating these needs at the National Competition will be taken into consideration.

LEVELS OF COMPETITION

There are four levels in the MATHCOUNTS Competition Series: school, chapter (local), state and national. Competition questions are written for 6th, 7th and 8th graders. The competitions can be quite challenging, particularly for students who have not been coached using MATHCOUNTS materials. All competition materials are prepared by the national office.

School Competitions (Ideally Held in January 2015): After several months of coaching, schools registered for the Competition Series should administer the 2015 School Competition to all interested students. The School Competition should be an aid to the coach in determining competitors for the Chapter Competition. *Selection of team and individual competitors is entirely at the discretion of the coach and does not need to be based solely on School Competition scores.* School Competition materials are sent to the coach of a school, and it may be used by the teachers and students only in association with that school's programs and activities. The current year's School Competition questions must remain confidential and may not be used in outside activities, such as tutoring sessions or enrichment programs with students from other schools. For updates or edits, please check www.mathcounts.org/competitioncoaches before administering the School Competition.

It is important that the coach look upon coaching sessions during the academic year as opportunities to develop better math skills in all students, not just in those students who will be competing. Therefore, it is suggested that the coach postpone selection of competitors until just prior to the Chapter Competition.

Chapter Competitions (Held from January 31 to February 28, 2015): The Chapter Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) may or may not be conducted. The chapter and state coordinators determine the date and location of the Chapter Competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The winning team will advance to the State Competition. Additionally, the two highest-ranking competitors not on the winning team (who may be registered as individuals or as members of a team) will advance to the State Competition. This is a minimum of six advancing Mathletes (assuming the winning team has four members). Additional teams and/or individuals also may progress at the discretion of the state coordinator, but the policy for progression must be consistent for all chapters within a state.

State Competitions (Held from March 1 to March 31, 2015): The State Competition consists of the Sprint, Target and Team Rounds. The Countdown Round (official or just for fun) may or may not be included. The state coordinator determines the date and location of the State Competition in accordance with established national procedures and rules. Winning teams and students will receive recognition. The four highest-ranked Mathletes and the coach of the winning team from each State Competition will receive an all-expenses-paid trip to the National Competition.

2015 Raytheon MATHCOUNTS National Competition (Held Friday, May 8, 2015 in Boston, MA): The National Competition consists of the Sprint, Target, Team and Countdown Rounds (conducted officially). Expenses of the state team and coach to travel to the National Competition will be paid by MATHCOUNTS. The national program does not make provisions for the attendance of additional students or coaches. All national competitors will receive a plaque and other items in recognition of their achievements. Winning teams and individuals also will receive medals, trophies and college scholarships.

COMPETITION COMPONENTS

The following four rounds of a MATHCOUNTS competition are designed to be completed in approximately three hours.

Target Round (approximately 30 minutes): In this round 8 problems are presented to competitors in four pairs (6 minutes per pair). The multistep problems featured in this round engage Mathletes in mathematical reasoning and problem-solving processes. *Problems assume the use of calculators.*

Sprint Round (40 minutes): Consisting of 30 problems, this round tests accuracy, with the time period allowing only the most capable students to complete all of the problems. *Calculators are not permitted.*

Team Round (20 minutes): In this round, interaction among team members is permitted and encouraged as they work together to solve 10 problems. *Problems assume the use of calculators.*

Note: The order in which the written rounds (Target, Sprint and Team) are administered is at the discretion of the competition coordinator.

Countdown Round: A fast-paced oral competition for top-scoring individuals (based on scores in the Target and Sprint Rounds), this round allows pairs of Mathletes to compete against each other and the clock to solve problems. *Calculators are not permitted.*

At Chapter and State Competitions, a Countdown Round (1) may be conducted officially, (2) may be conducted unofficially (for fun) or (3) may be omitted. However, the use of an official Countdown Round must be consistent for all chapters within a state. In other words, *all* chapters within a state must use the round officially in order for *any* chapter within a state to use it officially. All students, whether registered as part of a school team or as individual competitors, are eligible to qualify for the Countdown Round.

An official Countdown Round determines an individual's final overall rank in the competition. If a Countdown Round is used officially, the official procedures as established by the MATHCOUNTS Foundation must be followed, as described below.*

- The top 25% of students, up to a maximum of 10, are selected to compete. These students are chosen based on their Individual Scores.
- The two lowest-ranked students are paired; a question is read and projected, and students are given 45 seconds to solve the problem. A student may buzz in at any time, and if he or she answers correctly, a point is scored. If a student answers incorrectly, the other student has the remainder of the 45 seconds to answer.
- A total of three total questions are read to the pair of students, one question at a time, and the student who scores the higher number of points (not necessarily 2 out of 3) progresses to the next round and challenges the next-higher-ranked student.
- If students are tied in their matchup after three questions (at 1-1 or 0-0), questions should continue to be read until one is successfully answered. The first student who answers an additional question correctly progresses to the next round.
- This procedure continues until the fourth-ranked Mathlete and hisor her opponent compete. For the final four matchups, the first student to correctly answer three questions advances.
- The Countdown Round proceeds until a first place individual is identified. More details about Countdown Round procedures are included in the 2015 School Competition.

**Rules for the Countdown Round change for the National Competition.*

An unofficial Countdown Round does not determine an individual's final overall rank in the competition but is done for practice or for fun. The official procedures do not have to be followed. Chapters and states choosing not to conduct the round officially must determine individual winners solely on the basis of students' scores in the Target and Sprint Rounds of the competition.

SCORING

MATHCOUNTS Competition Series scores do not conform to traditional grading scales. Coaches and students should view an Individual Score of 23 (out of a possible 46) as highly commendable.

Individual Score: calculated by taking the sum of the number of Sprint Round questions answered correctly and twice the number of Target Round questions answered correctly. There are 30 questions in the Sprint Round and 8 questions in the Target Round, so the maximum possible Individual Score is $30 + 2(8) = 46$. If used officially, the Countdown Round yields final individual standings.

Team Score: calculated by dividing the sum of the team members' Individual Scores by 4 (even if the team has fewer than four members) and adding twice the number of Team Round questions answered correctly. The highest possible Individual Score is 46. Four students may compete on a team, and there are 10 questions in the Team Round. Therefore, the maximum possible Team Score is $((46 + 46 + 46 + 46) \div 4) + 2(10) = 66$.

Tiebreaking Algorithm: used to determine team and individual ranks and to determine which individuals qualify for the Countdown Round. In general, questions in the Target, Sprint and Team Rounds increase in difficulty so that the most difficult questions occur near the end of each round. In a comparison of questions to break ties, generally those who correctly answer the more difficult questions receive the higher rank. The guidelines provided below are very general; competition officials receive more detailed procedures.

- *Ties between individuals:* The student with the higher Sprint Round score will receive the higher rank. If a tie remains after this comparison, specific groups of questions from the Target and Sprint Rounds are compared.
- *Ties between teams:* The team with the higher Team Round score, and then the higher sum of the team members' Sprint Round scores, receives the higher rank. If a tie remains after these comparisons, specific questions from the Team Round will be compared.

RESULTS DISTRIBUTION

Coaches should expect to receive the scores of their students, as well as a list of the top 25% of students and top 40% of teams, from their competition coordinators. In addition, single copies of the blank competition materials and answer keys may be distributed to coaches after all competitions at that level nationwide have been completed. *Before distributing blank competition materials and answer keys, coordinators must wait for verification from the national office that all such competitions have been completed.* Both the problems and answers from Chapter and State Competitions will be posted on the MATHCOUNTS website following the completion of all competitions at that level nationwide, replacing the previous year's posted tests.

Student competition papers and answers will not be viewed by or distributed to coaches, parents, students or other individuals. Students' competition papers become the confidential property of MATHCOUNTS.

ADDITIONAL RULES

All answers must be legible.

Pencils and paper will be provided for Mathletes by competition organizers. However, students may bring their own pencils, pens and erasers if they wish. They may not use their own scratch paper or graph paper.

Use of notes or other reference materials (including dictionaries and translation dictionaries) is prohibited.

Specific instructions stated in a given problem take precedence over any general rule or procedure.

Communication with coaches is prohibited during rounds but is permitted during breaks. All communication between guests and Mathletes is prohibited during competition rounds. Communication between teammates is permitted only during the Team Round.

Calculators are not permitted in the Sprint and Countdown Rounds, but they are permitted in the Target, Team and Tiebreaker (if needed) Rounds. When calculators are permitted, students may use any calculator (including programmable and graphing calculators) that does not contain a QWERTY (typewriter-like) keypad. Calculators that have the ability to enter letters of the alphabet but do not have a keypad in a standard typewriter arrangement are acceptable. Smart phones, laptops, iPads®, iPods®, personal digital assistants (PDAs) and any other "smart" devices are not considered to be calculators and may not be used during competitions. Students may not use calculators to exchange information with another person or device during the competition.

Coaches are responsible for ensuring their students use acceptable calculators, and students are responsible for providing their own calculators. Coordinators are not responsible for providing Mathletes with calculators or batteries before or during MATHCOUNTS competitions. Coaches are strongly advised to bring backup calculators and spare batteries to the competition for their team members in case of a malfunctioning calculator or weak or dead batteries. Neither the MATHCOUNTS Foundation nor coordinators shall be responsible for the consequences of a calculator's malfunctioning.

Pagers, cell phones, iPods® and other MP3 players should not be brought into the competition room. Failure to comply could result in dismissal from the competition.

Should there be a rule violation or suspicion of irregularities, the MATHCOUNTS coordinator or competition official has the obligation and authority to exercise his or her judgment regarding the situation and take appropriate action, which might include disqualification of the suspected student(s) from the competition.

FORMS OF ANSWERS

The following rules explain acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Judges will score competition answers in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lb 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, $0.25 will not be accepted.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not have a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: What is 8 ÷ 12 expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: What is 12 ÷ 8 expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: What is the sum of the lengths of the radius and the circumference of a circle of diameter $\frac{1}{4}$ unit, expressed as a common fraction in terms of π? *Answer:* $\frac{1+2\pi}{8}$

Problem: What is 20 ÷ 12 expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Acceptable Simplified Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:
Problem: What is $\sqrt{15} \times \sqrt{5}$ expressed in simplest radical form? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form ($)*a.bc*, where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when *a* is zero, in which case it may be omitted, or when *b* and *c* are both zero, in which case they may both be omitted. Answers in the form ($)*a.bc* should be rounded to the nearest cent, unless otherwise specified. Examples:
Acceptable Forms: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not do any intermediate rounding (other than the "rounding" a calculator performs) when calculating solutions. All rounding should be done at the end of the calculation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \leq |a| < 10$, and n is an integer. Examples:
Problem: What is 6895 expressed in scientific notation? *Answer:* 6.895×10^3
Problem: What is 40,000 expressed in scientific notation? *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole-number answers should be expressed in their whole-number form. Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

VOCABULARY AND FORMULAS

The following list is representative of terminology used in the problems but should not be viewed as all-inclusive. It is recommended that coaches review this list with their Mathletes.

absolute difference
absolute value
acute angle
additive inverse *(opposite)*
adjacent angles
algorithm
alternate exterior angles
alternate interior angles
altitude *(height)*
apex
area
arithmetic mean
arithmetic sequence
base 10
binary
bisect
box-and-whisker plot
center
chord
circle
circumference
circumscribe
coefficient
collinear
combination
common denominator
common divisor
common factor
common fraction
common multiple
complementary angles
composite number
compound interest
concentric
cone
congruent
convex
coordinate plane/system
coordinates of a point
coplanar
corresponding angles
counting numbers
counting principle
cube
cylinder
decagon
decimal
degree measure
denominator
diagonal of a polygon
diagonal of a polyhedron
diameter
difference
digit
digit-sum
direct variation
dividend
divisible
divisor
dodecagon
dodecahedron
domain of a function
edge
endpoint
equation
equiangular
equidistant
equilateral
evaluate
expected value
exponent
expression
exterior angle of a polygon
factor
factorial
finite
formula
frequency distribution
frustum
function
GCF
geometric mean
geometric sequence
height *(altitude)*
hemisphere
heptagon
hexagon
hypotenuse
imageof a point
(under a transformation)
improper fraction
inequality
infinite series
inscribe
integer
interior angle of a polygon
interquartile range
intersection
inverse variation
irrational number
isosceles
kite
lateral edge
lateral surface area
lattice point(s)
LCM
linear equation
mean
median of a set of data
median of a triangle
midpoint
mixed number
mode(s) of a set of data
multiple
multiplicative inverse *(reciprocal)*
natural number
nonagon
numerator
obtuse angle
octagon
octahedron
odds (*probability)*
opposite of a number *(additive inverse)*
ordered pair
origin
palindrome
parallel
parallelogram
Pascal's Triangle
pentagon
percent increase/decrease
perimeter
permutation
perpendicular
planar
polygon
polyhedron
prime factorization

prime number
principal square root
prism
probability
product
proper divisor
proper factor
proper fraction
proportion
pyramid
Pythagorean Triple
quadrant
quadrilateral
quotient
radius
random
range of a data set
range of a function
rate
ratio
rational number
ray
real number
reciprocal *(multiplicative inverse)*
rectangle
reflection
regular polygon
relatively prime
remainder
repeating decimal
revolution
rhombus
right angle
right circular cone
right circular cylinder
right polyhedron
right triangle
rotation
scalene triangle
scientific notation
sector
segment of a circle
segment of a line
semicircle
semiperimeter
sequence
set
significant digits
similar figures
simple interest
slope
slope-intercept form
solution set
sphere
square
square root
stem-and-leaf plot
sum
supplementary angles
system of equations/inequalities
tangent figures
tangent line
term
terminating decimal
tetrahedron
total surface area
transformation
translation
trapezoid
triangle
triangular numbers
trisect
twin primes
union
unit fraction
variable
vertex
vertical angles
volume
whole number
x-axis
x-coordinate
x-intercept
y-axis
y-coordinate
y-intercept

The list of formulas below is representative of those needed to solve MATHCOUNTS problems but should not be viewed as the only formulas that may be used. Many other formulas that are useful in problem solving should be discovered and derived by Mathletes.

CIRCUMFERENCE

Circle	$C = 2 \times \pi \times r = \pi \times d$

AREA

Circle	$A = \pi \times r^2$
Square	$A = s^2$
Rectangle	$A = l \times w = b \times h$
Parallelogram	$A = b \times h$
Trapezoid	$A = \frac{1}{2}(b_1 + b_2) \times h$
Rhombus	$A = \frac{1}{2} \times d_1 \times d_2$
Triangle	$A = \frac{1}{2} \times b \times h$
Triangle	$A = \sqrt{s(s-a)(s-b)(s-c)}$
Equilateral triangle	$A = \frac{s^2\sqrt{3}}{4}$

SURFACE AREA AND VOLUME

Sphere	$SA = 4 \times \pi \times r^2$
Sphere	$V = \frac{4}{3} \times \pi \times r^3$
Rectangular prism	$V = l \times w \times h$
Circular cylinder	$V = \pi \times r^2 \times h$
Circular cone	$V = \frac{1}{3} \times \pi \times r^2 \times h$
Pyramid	$V = \frac{1}{3} \times B \times h$

Pythagorean Theorem	$c^2 = a^2 + b^2$
Counting/ Combinations	${}_nC_r = \frac{n!}{r!(n-r)!}$

ANSWERS

In addition to the answer, we have provided a difficulty rating for each problem. Our scale is 1-7, with 7 being the most difficult. These are only approximations, and how difficult a problem is for a particular student will vary. Below is a general guide to the ratings:

Difficulty 1/2/3 - One concept; one- to two-step solution; appropriate for students just starting the middle school curriculum.

4/5 - One or two concepts; multistep solution; knowledge of some middle school topics is necessary.

6/7 - Multiple and/or advanced concepts; multistep solution; knowledge of advanced middle school topics and/or problem-solving strategies is necessary.

Warm-Up 1

Answer	Difficulty		
1. 495	(1)	6. 26	(3)
2. 11	(2)	7. B & C	(4)
3. 70	(2)	8. 11,304	(2)
4. $3\frac{23}{24}$	(3)	9. 5/9	(4)
5. 16	(3)	10. 12	(4)

Warm-Up 2

Answer	Difficulty		
11. 4	(3)	16. 1000 or 1000.00	(3)
12. 5	(4)	17. 0	(2)
13. 8	(3)	18. (3, 3)	(4)
14. 50	(3)	19. 5/9	(4)
15. 14	(4)	20. $\sqrt{21}$	(4)

Workout 1

Answer	Difficulty		
21. 49	(2)	26. 17	(4)
22. 1.225×10^9	(3)	27. 174	(3)
23. 12	(4)	28. 0.04	(5)
24. 3.34	(3)	29. 5	(3)
25. 400	(4)	30. 1/6	(3)

Warm-Up 3

Answer	Difficulty		
31. 5	(2)	36. 234 or 234.00	(2)
32. 2	(2)	37. 10	(3)
33. 1*	(3)	38. 15	(3)
34. 6	(3)	39. 4.80	(3)
35. 7/36	(3)	40. 1/4	(3)

Warm-Up 4

Answer	Difficulty		
41. a^2	(3)	46. 5	(3)
42. 1/8	(3)	47. 3456	(3)
43. 37	(4)	48. 78	(5)
44. 20	(3)	49. 60	(3)
45. $1\frac{1}{2}$	(3)	50. 4	(3)

Workout 2

Answer	Difficulty		
51. 32	(5)	56. 2 or 2.00	(3)
52. 58	(2)	57. 6	(2)
53. 27.7	(4)	58. 89	(3)
54. 10	(5)	59. 2008	(3)
55. 25	(5)	60. 8	(4)

** The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.*

Warm-Up 5

Answer	Difficulty	Answer	Difficulty
61. 300 or 300.00	(3)	66. 60	(4)
62. 56	(4)	67. 2.4	(3)
63. 2015	(3)	68. 16	(4)
64. 0	(4)	69. 10	(4)
65. 16/15	(4)	70. 2015	(3)

Warm-Up 6

Answer	Difficulty	Answer	Difficulty
71. 11/6	(3)	76. 1459	(3)
72. 80	(4)	77. 3	(4)
73. $15\sqrt{3}$	(4)	78. 12	(3)
74. 810	(3)	79. 46	(3)
75. 9	(4)	80. 12	(3)

Workout 3

Answer	Difficulty	Answer	Difficulty
81. 0	(4)	86. 2	(3)
82. 51	(3)	87. 41	(2)
83. 0.5	(3)	88. 31	(4)
84. 2900	(4)	89. 460	(4)
85. 3	(4)	90. 7	(4)

Warm-Up 7

Answer	Difficulty	Answer	Difficulty
91. 72	(3)	96. 56	(3)
92. 18π	(4)	97. 3/2	(3)
93. 9	(3)	98. 54	(2)
94. 50	(3)	99. 2	(3)
95. 9/8	(3)	100. 45	(3)

Warm-Up 8

Answer	Difficulty	Answer	Difficulty
101. 9	(2)	106. 682	(3)
102. 12	(2)	107. 48	(3)
103. 85	(2)	108. 11	(3)
104. 35	(4)	109. 35,800 or 35,800.00	(3)
105. 5	(3)	110. 2	(4)

Workout 4

Answer	Difficulty	Answer	Difficulty
111. 79	(4)	116. 1353	(4)
112. 331,776	(5)	117. 32.2	(4)
113. 47	(2)	118. $72\sqrt{2}$	(4)
114. 6	(2)	119. 1207	(4)
115. 22.5	(3)	120. 6	(5)

Warm-Up 9

Answer	Difficulty		
121. 120	(3)	126. 1024	(4)
122. 10	(3)	127. 17	(2)
123. 400 or 400.00	(2)	128. 14,580 or 14,580.00	(4)
124. 2	(2)	129. 27	(3)
125. 5	(3)	130. 6	(3)

Warm-Up 10

Answer	Difficulty		
131. $64 + 32\pi$ or $32\pi + 64$	(4)	136. 27	(5)
132. 792	(3)	137. 2279	(5)
133. $\sqrt{3}/2$	(3)	138. 2	(3)
134. 30	(4)	139. 16	(4)
135. 4	(3)	140. 80	(4)

Workout 5

Answer	Difficulty		
141. 7.5	(3)	146. 2.3	(4)
142. 2.7	(5)	147. 8:12	(3)
143. 66,660	(5)	148. 2001	(4)
144. 22	(3)	149. 78	(3)
145. 0.3	(5)	150. 10	(3)

Warm-Up 11

Answer	Difficulty		
151. 32	(3)	156. 8/27	(5)
152. 60	(4)	157. (−3, 1)	(4)
153. 4/9	(4)	158. 16	(3)
154. $3\frac{1}{3}$	(4)	159. 12	(5)
155. 78	(4)	160. 648	(4)

Warm-Up 12

Answer	Difficulty		
161. 1024	(3)	166. 60	(5)
162. 34	(3)	167. 24	(5)
163. $2\sqrt{7}$	(5)	168. 1	(4)
164. 4/5	(4)	169. $144w^3$	(3)
165. 15	(3)	170. 9	(5)

Workout 6

Answer	Difficulty		
171. 76	(4)	176. 12	(3)
172. 251.6	(5)	177. 29	(5)
173. 1012	(5)	178. 4576	(4)
174. 3.4	(4)	179. 121/282	(5)
175. 33	(4)	180. 4	(4)

Warm-Up 13

Answer	Difficulty		
181. 36	(4)	186. 14	(3)
182. $\sqrt{5}/2$	(4)	187. 1/12	(4)
183. $2 + \sqrt{2}$ or $\sqrt{2} + 2$	(4)	188. 18	(3)
184. 25	(3)	189. −3/4	(3)
185. 17	(2)	190. 16	(4)

Warm-Up 14

Answer	Difficulty		
191. 324π	(4)	196. 3	(4)
192. 37/64	(4)	197. $4\frac{2}{7}$	(4)
193. 7/3	(4)	198. 6	(4)
194. 22	(3)	199. 1/6	(5)
195. 1008/2015	(5)	200. 757	(4)

Workout 7

Answer	Difficulty		
201. 25	(4)	206. 4.57	(4)
202. 13/55	(3)	207. 403	(3)
203. 46.1	(6)	208. 2	(4)
204. 42.7	(4)	209. 9	(3)
205. −5050	(4)	210. 12	(4)

Warm-Up 15

Answer	Difficulty		
211. 20	(4)	216. 1/100	(4)
212. 148	(5)	217. 61	(2)
213. −7	(3)	218. 55	(4)
214. $3 - \sqrt{3}$ or $-\sqrt{3} + 3$	(6)	219. 1 or 1.00	(3)
215. 60	(4)	220. 10	(4)

Warm-Up 16

Answer	Difficulty		
221. 0	(3)	226. 10	(4)
222. $6 + \pi$ or $\pi + 6$	(5)	227. 12,288	(3)
223. 14	(4)	228. 1358	(3)
224. 60	(4)	229. 10	(4)
225. 7/37	(5)	230. 55	(5)

Workout 8

Answer	Difficulty		
231. $\sqrt{6}$	(4)	236. 4.5	(5)
232. 7200 or 7200.00	(3)	237. −406	(4)
233. 0.155	(5)	238. 3.7	(4)
234. 84	(5)	239. 78.5	(5)
235. 0.18	(5)	240. 53	(4)

Warm-Up 17

Answer	Difficulty
241. 12	(5)
242. 12	(4)
243. $3 - \sqrt{3}$ or $-\sqrt{3} + 3$	(5)
244. $1 + \sqrt{3}$ or $\sqrt{3} + 1$	(5)
245. 40	(4)
246. 44	(4)
247. 19	(6)
248. 20	(5)
249. 5	(6)
250. 60	(4)

Warm-Up 18

Answer	Difficulty
251. $(47\sqrt{3})/2$	(5)
252. $1 + \sqrt{2}$ or $\sqrt{2} + 1$	(4)
253. 9	(6)
254. 20	(5)
255. 17/24	(4)
256. 1/27	(5)
257. 5/14	(5)
258. 11/140	(5)
259. 27	(5)
260. 75	(4)

Workout 9

Answer	Difficulty
261. 1	(4)
262. $c^2/(2ab)$	(4)
263. 32.9	(5)
264. 7/16	(6)
265. 0.049	(6)
266. 397.6	(4)
267. 76	(4)
268. 14.4	(4)
269. 6	(5)
270. 45	(4)

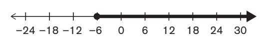
Logic Stretch

Answer	Difficulty
271. 7	(2)
272. 15	(3)
273. 9	(3)
274. 2	(1)
275. 12	(3)
276. 1038	(3)
277. 39	(3)
278. 8	(2)
279. 3/7	(4)
280. 972	(4)

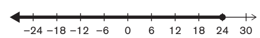
Solving Inequalities Stretch

Answer **Difficulty**

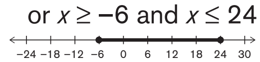
281. $x \geq -6$ (3)



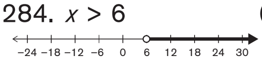
282. $x \leq 24$ (3)



283. $-6 \leq x \leq 24$ (4)
or $x \geq -6$ and $x \leq 24$

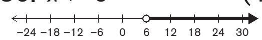


284. $x > 6$ (3)

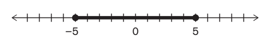


285. $x > 3$ (3)

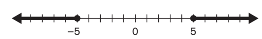
286. $x > 6$ (4)



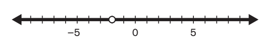
287. $-5 \leq x \leq 5$ (5)
or $x \geq -5$ and $x \leq 5$



288. $x \leq -5$ or $x \geq 5$ (5)



289. $x \neq -2$ (6)
or $x < -2$ or $x > -2$



290. $x < -3$ or $x > -1$ (6)

Mass Point Geometry Stretch

Answer	Difficulty
291. 5/2	(5)
292. 7/3	(5)
293. 1/4	(5)
294. 1/2	(5)
295. 4/25	(6)
296. 1/4	(6)
297. 60	(6)
298. 16/105	(7)
299. 25/38	(7)
300. 2/5	(7)

MATHCOUNTS Problems Mapped to Common Core State Standards (CCSS)

Forty-three states, the District of Columbia, four territories and the Department of Defense Education Activity (DoDEA) have adopted the Common Core State Standards (CCSS). As such, MATHCOUNTS considers it beneficial for teachers to see the connections between the *2014-2015 MATHCOUNTS School Handbook* problems and the CCSS. MATHCOUNTS not only has identified a general topic and assigned a difficulty level for each problem but also has provided a CCSS code in the Problem Index (pages 83-84). A complete list of the Common Core State Standards can be found at www.corestandards.org.

The CCSS for mathematics cover K-8 and high school courses. MATHCOUNTS problems are written to align with the NCTM Standards for Grades 6-8. As one would expect, there is great overlap between the two sets of standards. MATHCOUNTS also recognizes that in many school districts, algebra and geometry are taught in middle school, so some MATHCOUNTS problems also require skills taught in those courses.

In referring to the CCSS, the Problem Index code for each or the Standards for Mathematical Content for grades K-8 begins with the grade level. For the Standards for Mathematical Content for high school courses (such as algebra or geometry), each code begins with a letter to indicate the course name. The second part of each code indicates the domain within the grade level or course. Finally, the number of the individual standard within that domain follows. Here are two examples:

- *6.RP.3 → Standard #3 in the Ratios and Proportional Relationships domain of grade 6*
- *G-SRT.6 → Standard #6 in the Similarity, Right Triangles and Trigonometry domain of Geometry*

Some math concepts utilized in MATHCOUNTS problems are not specifically mentioned in the CCSS. Two examples are the Fundamental Counting Principle (FCP) and special right triangles. In cases like these, if a related standard could be identified, a code for that standard was used. For example, problems using the FCP were coded 7.SP.8, S-CP.8 or S-CP.9 depending on the context of the problem; SP → Statistics and Probability (the domain), S → Statistics and Probability (the course) and CP → Conditional Probability and the Rules of Probability. Problems based on special right triangles were given the code G-SRT.5 or G-SRT.6, explained above.

There are some MATHCOUNTS problems that either are based on math concepts outside the scope of the CCSS or based on concepts in the standards for grades K-5 but are obviously more difficult than a grade K-5 problem. When appropriate, these problems were given the code SMP for Standards for Mathematical Practice. The CCSS include the Standards for Mathematical Practice along with the Standards for Mathematical Content. The SMPs are (1) Make sense of problems and persevere in solving them; (2) Reason abstractly and quantitatively; (3) Construct viable arguments and critique the reasoning of others; (4) Model with mathematics; (5) Use appropriate tools strategically; (6) Attend to precision; (7) Look for and make use of structure and (8) Look for and express regularity in repeated reasoning.

PROBLEM INDEX

It is difficult to categorize many of the problems in the *MATHCOUNTS School Handbook*. It is very common for a MATHCOUNTS problem to straddle multiple categories and cover several concepts. This index is intended to be a helpful resource, but since each problem has been placed in exactly one category and mapped to exactly one Common Core State Standard (CCSS), the index is not perfect. In this index, the code **9 (3) 7.SP.3** refers to problem 9 with difficulty rating 3 mapped to CCSS 7.SP.3. For an explanation of the difficulty ratings refer to page 78. For an explanation of the CCSS codes refer to page 82.

Algebraic Expressions & Equations

2	(2)	SMP
13	(3)	7.EE.4
26	(4)	6.EE.7
33	(3)	8.EE.8
36	(2)	8.EE.8
41	(3)	6.EE.9
46	(3)	F-IF.2
50	(3)	6.EE.2
51	(5)	8.EE.8
64	(4)	SMP
68	(4)	8.EE.8
69	(4)	8.EE.8
70	(3)	SMP
81	(4)	8.EE.2
86	(3)	8.EE.8
93	(3)	8.EE.8
107	(3)	6.EE.2
108	(3)	8.EE.8
110	(4)	6.EE.2
112	(5)	SMP
123	(2)	6.EE.7
124	(2)	F-IF.2
125	(3)	A-SSE.2
127	(2)	6.EE.9
135	(3)	8.EE.8
144	(3)	6.EE.7
147	(3)	6.EE.9
149	(3)	8.EE.8
150	(3)	6.EE.9
154	(4)	8.EE.8
158	(3)	4.OA.4
176	(3)	7.NS.3
189	(3)	8.EE.8
193	(4)	F-IF.2
237	(4)	A-REI.4
240	(4)	A-CED.1
245	(4)	8.EE.2
281	(3)	7.EE.4
282	(3)	7.EE.4
283	(4)	A-CED.1
284	(3)	7.EE.4
285	(3)	7.EE.4
286	(4)	A-CED.1
287	(5)	A-CED.1
288	(5)	A-CED.1
289	(6)	A-CED.1
290	(6)	A-CED.1

General Math

1	(2)	5.MD.1
17	(2)	4.OA.3
161	(3)	7.NS.2
168	(4)	SMP
180	(4)	6.EE.2
196	(4)	F-IF.1
213	(3)	F-IF.2

Number Theory

8	(1)	4.OA.5
21	(2)	4.OA.4
31	(2)	SMP
32	(2)	SMP
37	(3)	SMP
47	(3)	6.NS.4
48	(5)	S-CP.9
57	(2)	7.NS.3
62	(4)	S-CP.9
63	(3)	4.OA.3
76	(3)	SMP
87	(2)	4.OA.4
96	(3)	SMP
120	(5)	SMP
129	(3)	4.OA.4
138	(3)	7.NS.1
148	(4)	SMP
152	(4)	SMP
162	(3)	SMP
165	(3)	SMP
175	(4)	S-CP.9
181	(4)	SMP
184	(3)	SMP
190	(4)	8.EE.2
198	(4)	SMP
200	(4)	SMP
201	(4)	N-RN.2
202	(3)	7.NS.3
207	(3)	SMP
208	(4)	7.NS.2
209	(3)	7.NS.3
210	(4)	SMP
221	(3)	SMP
231	(4)	N-RN.1
254	(5)	SMP
255	(4)	N-RN.2
261	(4)	8.EE.2
270	(4)	7.NS.3

Statistics

14	(3)	SMP
60	(4)	6.SP.5
199	(5)	7.SP.8
235	(5)	7.SP.8
4	(3)	6.SP.2
12	(4)	6.SP.5
85	(4)	6.SP.2
91	(3)	6.SP.2
105	(3)	6.SP.2
122	(3)	6.SP.5
141	(3)	6.SP.2

Logic

7	(4)	SMP
132	(3)	SMP
271	(2)	SMP
272	(3)	SMP
273	(3)	SMP
274	(1)	SMP
275	(3)	SMP
276	(3)	SMP
277	(3)	SMP
278	(2)	SMP
279	(4)	SMP
280	(4)	SMP

Problem Solving (Misc.)

6	(3)	SMP
43	(4)	SMP
44	(3)	SMP
55	(5)	SMP
58	(3)	SMP
90	(4)	SMP
99	(3)	SMP
104	(4)	SMP
116	(4)	SMP
166	(5)	6.EE.9
172	(5)	G-C.2
192	(4)	7.G.6
195	(5)	SMP
218	(4)	SMP
230	(5)	SMP
234	(5)	SMP
242	(4)	SMP
246	(4)	SMP
259	(5)	SMP

Measurement

3	(2)	6.RP.3
15	(4)	6.RP.3
22	(3)	8.EE.4
38	(3)	6.RP.3
39	(3)	6.RP.3
23	(4)	7.G.4
28	(5)	7.G.4
40	(3)	7.G.6
111	(4)	8.G.7
117	(4)	8.G.7
126	(4)	G-SRT.6
131	(4)	7.G.6
133	(3)	G-SRT.6
139	(4)	G-SRT.6
146	(4)	G-SRT.6
159	(5)	G-SRT.6
167	(5)	G-SRT.6
174	(4)	G-SRT.6
177	(5)	8.G.7
182	(4)	8.G.7
191	(4)	8.G.7
206	(4)	8.G.7
214	(6)	G-SRT.6
233	(5)	G-SRT.6
243	(5)	G-SRT.6
244	(5)	G-SRT.6
247	(6)	G-SRT.6
251	(5)	G-SRT.6
263	(5)	G-SRT.6
269	(5)	8.G.7

Coordinate Geometry

18	(4)	8.EE.8
29	(3)	8.EE.8
49	(3)	6.G.3
100	(3)	8.F.3
157	(4)	8.F.3
204	(4)	F-IF.2
220	(4)	8.G.8
228	(3)	8.F.3
238	(4)	G-SRT.6
239	(5)	SMP
262	(4)	8.F.3

Sequences, Series & Patterns

34	(3)	F-BF.2
71	(3)	F-BF.2
95	(3)	F-BF.2
103	(2)	F-BF.2
114	(2)	F-BF.2
137	(5)	F-BF.2
170	(5)	F-BF.2
171	(4)	SMP
205	(4)	F-BF.2
229	(4)	F-BF.2
241	(5)	F-BF.2

Plane Geometry

9	(4)	8.G.7
20	(4)	8.G.7
53	(4)	7.G.4
75	(4)	SMP
92	(4)	7.G.4
101	(2)	SMP
121	(3)	8.G.5
136	(5)	G-CO.10
145	(5)	G-C.2
163	(5)	G-C.2
212	(5)	G-C.2
222	(5)	7.G.4
223	(4)	G-C.2
225	(5)	G-GMD.3
248	(5)	G-C.2
249	(6)	G-C.2
252	(4)	8.G.7
253	(6)	8.G.7
260	(4)	G-C.2

Solid Geometry

11	(3)	G-GMD.3
77	(4)	G-GMD.3
79	(3)	7.G.6
88	(4)	7.G.6
106	(3)	7.G.6
118	(4)	8.G.9
119	(4)	7.G.6
130	(3)	SMP
134	(4)	G-GMD.3
155	(4)	7.G.6
169	(3)	6.G.2
178	(4)	7.G.6
194	(3)	7.G.6
224	(4)	SMP
226	(4)	SMP
227	(3)	G-GMD.3
236	(5)	G-GMD.3
256	(5)	SMP
266	(4)	G-GMD.3
268	(4)	7.G.4

Probability, Counting & Combinatorics

5	(3)	7.SP.8
19	(4)	7.SP.8
30	(3)	7.SP.8
35	(3)	7.SP.8
42	(3)	7.SP.8
78	(3)	7.SP.8
80	(3)	SMP
143	(5)	S-CP.9
151	(3)	SMP
153	(4)	S-CP.8
160	(4)	S-CP.8
179	(5)	7.SP.8
187	(4)	7.SP.8
211	(4)	S-CP.9
216	(4)	7.SP.8
232	(3)	S-CP.9
258	(5)	7.SP.8
264	(6)	7.SP.8
265	(6)	7.SP.8

Percents & Fractions

10	(4)	6.RP.3
16	(3)	7.RP.3
24	(3)	7.RP.3
25	(4)	6.RP.3
27	(3)	6.RP.3
52	(2)	6.RP.3
56	(3)	7.RP.3
59	(3)	6.RP.3
65	(4)	6.RP.3
66	(4)	6.RP.3
67	(3)	6.RP.3
72	(4)	7.RP.3
74	(3)	6.RP.3
82	(3)	7.RP.3
89	(4)	7.RP.3
94	(3)	6.RP.3
98	(2)	6.RP.3
102	(2)	6.NS.1
109	(3)	7.RP.3
128	(4)	7.RP.3
140	(4)	6.RP.3
188	(3)	6.RP.3
215	(4)	7.RP.3
217	(2)	7.RP.3
257	(5)	7.RP.3
267	(4)	6.RP.3

Proportional Reasoning

54	(5)	7.G.6
73	(4)	7.G.6
83	(3)	7.G.6
97	(3)	6.G.1
113	(2)	7.G.6
142	(5)	7.RP.2
156	(5)	7.G.6
164	(4)	G-SRT.5
183	(4)	6.G.1
197	(4)	G-SRT.5
203	(6)	7.G.6
291	(5)	6.RP.1
292	(5)	6.RP.1
293	(5)	6.RP.1
294	(5)	6.RP.1
295	(6)	6.RP.1
296	(6)	6.RP.1
297	(6)	6.RP.1
298	(7)	6.RP.1
299	(7)	6.RP.1
300	(7)	6.RP.1
45	(3)	6.RP.3
61	(3)	6.RP.3
84	(4)	6.RP.3
115	(3)	6.RP.3
173	(5)	G-C.2
185	(2)	6.RP.3
186	(3)	6.RP.3
219	(3)	6.RP.3
250	(4)	6.EE.7

MATHCOUNTS® COMPETITION SERIES

2014–2015 ADDITIONAL STUDENTS REGISTRATION FORM

Step 1: ***Tell us about your school so we can find your original registration (please print legibly).***

Teacher/Coach Name ______________________ Teacher/Coach Email ______________________

School Name ______________________ Teacher/Coach Phone ______________________

School Mailing Address ______________________ City, State ZIP ______________________

Step 2: ***Tell us how many students you are adding to your school's registration. Following the instructions below.***

Please circle *the number of additional students you will enter in the Chapter Competition and the associated cost below (depending on the date your registration is postmarked). The cost is $30 per student added, whether that student will be part of a team or will compete as an individual. The cost of adding students to a previous registration is not eligible for an Early Bird rate.*

# of Students You Are Adding	1	2	3	4	5	6	7	8	9
Regular Rate *(postmarked **by** Dec. 12, 2014)*	$30	$60	$90	$120	$150	$180	$210	$240	$270
Late Registration *(postmarked **after** Dec. 12, 2014)*	$50	$80	$110	$140	$170	$200	$230	$260	$290

☐ My school qualifies for the 50% Title I discount, so the Amount Due in Step 4 will be half the amount I circled above. Principal signature required to verify Title I eligibility.

X ______________________

Step 3: ***Tell us what your school's FINAL registration should be (including all changes/additions).***

TOTAL # of Registered Students	1 (1 individual)	2 (2 ind)	3 (3 ind)	4 (1 team)	5 (1 tm, 1 ind)	6 (1 tm, 2 ind)	7 (1 tm, 3 ind)	8 (1 tm, 4 ind)	9 (1 tm, 5 ind)	10 (1 tm, 6 ind)

Step 4: ***Almost done... just fill in payment information and turn in your form!***

Amount Due __________ ☐ Check *(payable to MATHCOUNTS Foundation)* ☐ Money order ☐ Purchase order # __________ *(must include P.O.)*

☐ Credit card *(include all information)* Name on card ______________________ ☐ Visa ☐ MasterCard

Signature ______________________ Card # ______________________ Exp. __________

Mail, email a scanned copy or fax this completed form to:

MATHCOUNTS Registration | P.O. Box 441 | Annapolis Junction, MD 20701

Email: reg@mathcounts.org | Fax: 240-396-5602

Questions?
Please call the Registration Office at ***301-498-6141.***

Registration is fast and easy at www.mathcounts.org/clubreg!
Get an instant email confirmation without having to wait for processing.

THE NATIONAL **MATHCLUB** powered by MATHCOUNTS®

2014–2015 REGISTRATION FORM

Step 1: *Tell us about your group. Check and complete only 1 option.*

☐ ***U.S. school with students in 6th, 7th and/or 8th grade***

School Name: ____________________

There can be multiple clubs at the same U.S. middle school, as long as each club has a different Club Leader.

☐ ***Chapter or member group of a larger organization.***
(Can be non-profit or for profit)

Organization: ____________________

Chapter (or equivalent) Name: ____________________

Examples of larger organizations: Girl Scouts, Boy Scouts, YMCA, Boys & Girls Club, nationwide tutoring/enrichment centers.

☐ ***A home school or group of students not affiliated with a larger organization.***

Club Name: ____________________

Examples: home schools, neighborhood math groups, independent tutoring centers

Step 2: *Make sure your group is eligible to participate in The National Math Club.*

Please check off that all 3 of the following statements are true for your group, to the best of your knowledge:

☐ My group consists of at least 4 U.S. students.
☐ The students in my group are in 6th, 7th and/or 8th grade.
☐ My group has regular in-person meetings.

By signing below I, the Club Leader, affirm that all of the above statements are true, to the best of my knowledge, and that my group is therefore eligible to participate in The National Math Club. I understand that MATHCOUNTS can cancel my membership at any time if it is determined that my group is ineligible.

Club Leader Signature: ____________________

Step 3: *Get signed up.*

Club Leader Name ____________________ Club Leader Phone ____________________

Club Leader Email ____________________

Club Leader Alternate Email ____________________

Club Mailing Address ____________________

City, State ZIP ____________________

Total # of participating students in club: ________ ☐ Previously participated in MATHCOUNTS.

How did you hear about MATHCOUNTS? ☐ Mailing ☐ Word-of-mouth ☐ Conference ☐ Internet ☐ Email ☐ Prior Participant

For schools only: School Type: ☐ Public ☐ Charter ☐ Private ☐ Home school ☐ Virtual

If your school is overseas: My school is sponsored by ☐ DoDDS. OR ☐ State Department. Country ____________________

Step 4: *Almost done... just turn in your form.*

Mail, email a scanned copy or fax this completed form to:
MATHCOUNTS Foundation, 1420 King Street, Alexandria, VA 22314
Email: info@mathcounts.org
Fax: 703–299–5009

Questions?
Please call the national office at
703-299-9006