MATHCOUNTS®

The Fundamental Counting Principle

Warm-Up!

Coach instructions:
Give students around 10 minutes to go through the warm-up problems.

Try these problems before watching the lesson.

1. To make an ice cream sundae, Shelby chooses one scoop of chocolate, vanilla or strawberry ice cream and tops it with either sprinkles or whipped cream. How many different sundaes can she make?

   If we make a list of possible sundaes with one flavor and one toping, then we have following possible sundaes: chocolate & sprinkles, vanilla & sprinkles, strawberry & sprinkles, chocolate & whipped cream, vanilla & whipped cream and strawberry & whipped cream. This gives us 6 different sundaes. Alternatively, you could multiply 3 × 2 = 6 to get your answer using The Fundamental Counting Principle

2. How many different outcomes are possible when Tanya flips two coins, a dime and a quarter?

   There are 2 possible flips for the dime, heads or tails, and also 2 possible flips for the quarter. The two coins can land 2 × 2 = 4 different ways. Listing these possible flips out we get: H_DH_Q, H_DT_Q, T_DH_Q, T_DT_Q.

3. How many different seating arrangements are possible if Kendra, Mary and Darla reserved the first three seats in the last row of the movie theatre?

   Using their first initial to list seating arrangements, we get: KMD, KDM, MKD, MDK, DKM and DMK or 6 seating arrangements. Alternatively, there are 3 choices for the first seat, then 2 for the second seat and 1 for the last seat or 3 × 2 × 1 = 6.

4. How many different outfits, each consisting of a sweater and a pair of jeans, can Jelena make choosing from four sweaters and three pairs of jeans?

   There are 4 possible sweater choices and 3 possible jean choices or 4 × 3 = 12 different outfits. If we wanted to list them out, we could label the sweaters A, B, C and D and the jeans 1, 2 and 3 to get the following combinations: A1, A2, A3, B1, B2, B3, C1, C2, C3, D1, D2, D3 and D4.

The Problems

Coach instructions: *After students try the warm-up problems, play the video and have them follow along with the solutions.*

Take a look at the following problems and follow along as they are explained in the video.

5. Manny has 5 shirts, 3 pairs of pants, 2 ties and 4 pairs of shoes. If Manny's school uniform consists of a shirt, a pair of pants, a tie and a pair of shoes, how many different uniforms can he wear to school?

   Solution in video. Answer: 120 different uniforms.

   This video uses The Fundamental Counting Principle to solve the problem. The Fundamental Counting Principles is a way of counting the number of possible outcomes (or combinations) of a certain event. ***In general, The Fundamental Counting Principle tells us that if there are m ways to do one thing and n ways to do another, then there are m × n ways of doing both.*** **This principle will be helpful in counting problems, such as the ones to follow, and also in probabilitiy problems to figure out the number of favorable and/or total outcomes.**

Piece It Together

Coach instructions: *After watching the video, give students 10 to 15 minutes to try the next five problems.*

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

6. If Molly can choose from 5 kinds of fruit, 3 salads and 4 beverages for her lunch, how many different combinations of a fruit, a salad and a beverage can she make?

   Five choices for the fruit, three choices for the salad and four choices for Molly's beverage make a total of 5 × 3 × 4 = 60 combinations.

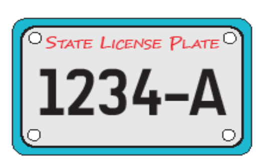
7. Bob has stencils to paint the digits 2, 5 and 8. How many distinct three digit house numbers can he paint, using only the stencils?

   Bob has three options for each of the three digits in the house number, meaning he can make 3 × 3 × 3 = 27 house numbers.

8. A deli specializes in gourment sandwiches. Each sandwich has one type of bread, one type of meat and possibly a condiment. The choices for bread are wheat, white, rye or poppyseed. The choices for meat are ham, salami or turkey. Finally, the sandwich can have mayonnaise, mustard or no condiment. How many different sandwiches can this deli make?

   For each complete sandwich, we need to know the type of bread, the type of meat and the type, if any, of condiment. We have 4 bread options, 3 meat options and 3 condiment options (mayo, mustard or nothing). In total, there are 4 × 3 × 3 = 36 different sandwiches.

9. License plates are issued that contain four digits followed by one letter. If the letters O and I cannot be used, how many different license plates are possible?



For each of the four numbers in the license plate, we have 10 choices — 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9. For the letter, we have 24 choices — the 26 letters of the alphabet minus O and I. This means we have 10 × 10 × 10 × 10 × 24 = 240,000 possible license plates.

10. A flag is to be designed using 3 differently colored vertical stripes. If 5 colors are available, how many distinct flags are possible?

For the first stripe of the flag, we have 5 choices. For the second stripe of the flag, we only have 4 choices because we need to use a different color so we cannot use whatever color was chosen for the first stripe. 60 combinations.

Optional Extension

Coach instructions: *Once your students have completed the problems and feel they have a comfortable understanding of the concept, have them stretch their thinking to these two problems with additional criteria/restrictions.*

To extend your understanding and have a little fun with math, try the following activities.

Now that you have applied The Fundamental Counting Principle to problems asking for the number of combinations for one type of event, try thinking about how to use it in a problem with multiple possible event outcomes. For example, look back at problem number 7 which asked how many distinct three digit house numbers can Bob paint using only the three stencils 2, 5 and 8. How might you change your application of The Fundamental Counting Principle to solve the problem if we changed it to read:

Bob has stencils to paint the digits 2, 5 and 8. How many distinct, one, two or three digit house numbers can he paint, using only the stencils?

For this problem, you can still use The Fundamental Counting Principle, but there are three separate cases to consider. In problem 7, students calculated that there are 27 house number consisting of three digits that can be made using these stencils. In this problem there are also 3 × 3 = 9 house numbers consisting of two digits that can be made and 3 house numbers that can be made with only one digit. To find our answer, we should add the three cases to get 27 + 9 + 3 = 39 house numbers.

If you are looking for another challenge to your counting abilities, try the following problem which adds some constraints to the possible outcomes:

Lily is going to the movies with Abby, Bea and Jacly. Abby wants to sit at the end of the row, and Bea only cares that she is seated next to Jaclyn. In how many different ways can the girls be seated in a single row that has only four seats?

There are 2 possible ends for Abby (A), 2 ways that Bea (B) and Jaclyn (J) can be next to each other and 2 choices for Lily (L) - either on the other end or between Abby and the pair of girls, Bea and Jaclyn. Here are the 2 × 2 × 2 = 8 ways they all can be seated: ABJL, AJBL, ALBJ, ALJB, BJLA, JBLA, LBJA, LJBA.

This problem also has a great video solution you can view here:
www.mathcounts.org/resources/video-library/math-video-challenge/back-good-old-days