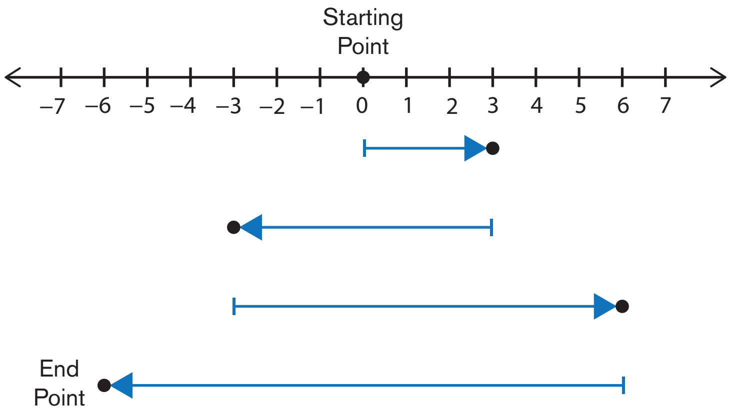
February 2010 Activity Solutions

Warm-Up!

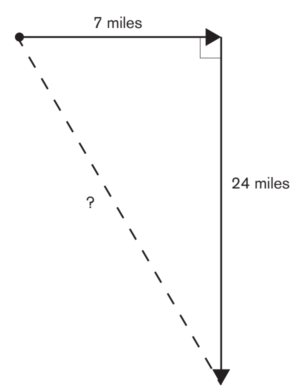
1. With three selections from these cards, you can obtain a sum of 3 with either 2, 1 and 0 or 1, 1 and 1. There are 3! = 3 × 2 × 1 = 6 ways to select 2, 1 and 0: (1, 2, 0), (1, 0, 2), (0, 1, 2), (0, 2, 1), (2, 1, 0), (2, 0, 1). There is only 1 way to select 1, 1 and 1, so there are 6 + 1 = **7** ways to choose the cards such that the sum of the numbers is 3.

2. With 0 and 1 being the only choices, in order to get a sum of 2, you must draw exactly two 1s (and therefore, two 0s as well). There are **6** ways to draw two 1s and two 0s: (1, 1, 0, 0), (1, 0, 1, 0), (1, 0, 0, 1), (0, 1, 1, 0), (0, 1, 0, 1), (0, 0, 1, 1).

3. Consider a number line. Let's say our starting point is at 0. Going 3 miles east puts us at 3 on the number line. From there, 6 miles west puts us at 3 – 6 = –3. Going 9 miles east from this point puts us at –3 + 9 = 6. Finally, going 12 miles west from here puts us at 6 – 12 = –6. Therefore, we are 0 – (–6) = **6** miles from where we started.



4. East and south are perpendicular directions, so as shown in the figure, we are looking for the length of the hypotenuse of a right triangle. We can use the Pythagorean theorem to solve: $x^2 = 7^2 + 24^2$, where x is the length of the hypotenuse in miles. Simplifying, we get $x^2 = 49 + 576 = 625$. Finally, taking the square root of both sides of the equation gives $x = 25$. So, we are **25** miles from where we started. (You may have also known that 7-24-25 is a Pythagorean Triple!)



The Problems are solved in the MATHCOUNTS® Mini video.

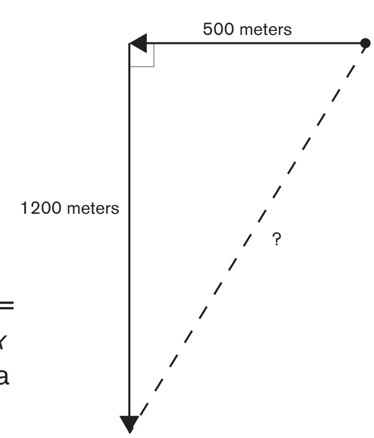
Follow-up Problems

5. If Alli can only earn 0, 1 or 2 points, then the only ways to get a sum of 3 points with her four oranges are 1 + 1 + 1 + 0 = 3 and 2 + 1 + 0 + 0 = 3. We must consider the arrangements of these possibilities to get the total number of possible scoring sequences. Let's start with 1, 1, 1 and 0. Alli could get the 0 points on any of the first, second, third or fourth launch, with the rest of the hits earning her 1 point each. Therefore, there are 4 ways in which Alli could earn 3 points with this point breakdown. Next, let's consider 2, 1, 0 and 0 points. Say Alli gets 2 points on her first hit. She could get 1 point with any of the remaining three launches, and then the other two launches would earn her 0 points each. So, there are 3 ways in which Alli could earn 3 points with 2 points from her first hit. Following this same reasoning, we find that there are 3 ways this could happen if Alli earns 2 points with her second hit; 3 ways this could happen if Alli earns 2 points with her third hit; and 3 ways this could happen if Alli earns 2 points with her fourth hit. Therefore, with this point breakdown, Alli could earn 3 points in 3 × 4 = 12 different ways. So, Alli could earn 3 points with 12 + 4 = **16** different scoring sequences.

6. There are only two ways to get 5 points with four oranges: 2 + 1 + 1 + 1 = 5 and 2 + 2 + 1 + 0 = 5. We must consider the arrangements of these possibilities to get the total number of possible scoring sequences. Let's start with 2, 1, 1 and 1. Alli could get 2 points on any of the first, second, third or fourth launch, with the rest of the hits earning her 1 point each. Therefore, there are 4 ways in which Alli could earn 5 points with this point breakdown. Next, let's consider 2, 2, 1 and 0. Say Alli gets 0 points on her first launch. She could get 1 point with any of the remaining three launches, and then the other two launches would earn her 2 points each. So, there are 3 ways in which Alli could earn 5 points with 0 points from her first launch. Following this same reasoning, we find that there are 3 ways this could happen if Alli earns 0 points with her second launch; 3 ways this could happen if Alli earns 0 points with her third launch; and 3 ways this could happen if Alli earns 0 points with her fourth launch. Therefore, with this point breakdown, Alli could earn 5 points in 3 × 4 = 12 different ways. So, Alli could earn 5 points with 12 + 4 = **16** different scoring sequences.

7. The answers to Problems 5 and 6 are the same because the first combination of points is of the form ABBB, while the second is of the form AABC in both problems. As a result, the number of possible arrangements of each is the same.

8. Digging straight down from the surface implies a perpendicular direction, so we are looking to find the length of the hypotenuse of a right triangle. This right triangle has one leg of length 500 meters (the distance along the surface of the land from the original location, as found in Problem 2 in the video) and one leg of length 1200 meters (digging downward), as shown. So, we can use the Pythagorean theorem to solve: $x^2 = 500^2 + 1200^2$, where x is the length of the hypotenuse in meters. Simplifying, we get $x^2 = 250{,}000 + 1{,}440{,}000 = 1{,}690{,}000$. Taking the square root of both sides of the equation gives $x = 1300$. Thus, the treasure is **1300** meters from where Prella and Boda started.



9. The solid black lines in the figure show the new path when “west” is changed to “northwest” and “east” is changed to “southeast.” Because “northwest” and “southeast” fall exactly between perpendicular directions (north and west; south and east, respectively), we know that both the triangle that contains point P and the triangle that contains point Q are 45-45-90 triangles. By laws of 45-45-90 triangles, we know that the lengths of the legs in the triangle containing point P are each $150\sqrt{2}$ meters, and the lengths of the legs in the triangle containing point Q are each $300\sqrt{2}$ meters. Because the hypotenuse of the triangle containing point P is half the length of the hypotenuse of the triangle containing point Q, we know that the distance between point E and the axis containing the starting point is also $150\sqrt{2}$ meters. To find the length of R, we take the distance from P to Q and subtract the vertical distance from P to E, or $R = 1000 + 300\sqrt{2} - (600 + 150\sqrt{2}) = 400 + 150\sqrt{2}$. These two lengths form the legs of a right triangle, of which the hypotenuse is the distance we need to find. So, we can use the Pythagorean theorem to solve: $x^2 = (150\sqrt{2})^2 + (400 + 150\sqrt{2})^2$, where x is the length of the missing hypotenuse. Simplifying, we get $x^2 = 150^2 \times 2 + 400^2 + 400 \times 150 \times 2\sqrt{2} + 150^2 \times 2 = 300^2 + 400^2 + 400 \times 300\sqrt{2} = 500^2 + 400 \times 300\sqrt{2} = 5^2 \times 10^4 + 12 \times 10^4\sqrt{2} = 10^4(25 + 12\sqrt{2})$. Taking the square root of both sides of the equation, we get that the distance from the starting point to the treasure is $100\sqrt{25+12\sqrt{2}}$ meters.